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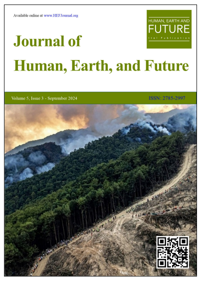

A Novel View on Dental Service Management Optimization Using Markov Processes

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Abstract

To develop a mathematical model for managing dental services as a Markov random process based on the state graph of the process of providing dental care to the population. Markov chains, limiting probabilities of states and equations of A.N. Kolmogorov, and a graph of possible states of the dental care provision process, enabling to mathematically describe and optimize the process of managing dental services using a Markov random process in Moscow. With a full return of patients for correction of defects in the same dental medical organization where the poor-quality service was provided and a maximum warranty period of 5 years, the average financial result (gross profit) of the considered dental medical organization and the relative average increase in the financial result are maximum and equal to Rub 5,018,18 and 64.42%, respectively. It is shown that with a slight increase in the warranty period from one to three years, the financial result (gross profit) grows faster than with a warranty period of 3 to 5 years. To achieve a relative average increase in financial results by a factor of 1.5, the warranty period for dental services provided should be 3.13 years.

Keywords: Mathematical Model; Markov Chains; Kolmogorov Equations; Limiting Probabilities.

1. Introduction

1.1. Brief Information About the Current State of Dental Services Management in Moscow and the Russian Federation as a Whole

Information technologies are widely introduced into practical and scientific dentistry. Informatization has significantly affected orthopedic dentistry in the fields of construction (modeling) of denture structures in the CAD system, analysis of predicting their durability in an environment of almost constant alternating loads in the CAE system, and digital production in the CAM system for computer milling and three-dimensional printing. The integration of traditional (analog) and digital techniques has formed a hybrid technology [1-3]. However, along with obvious clinical effectiveness, the question arises about the economic feasibility of such changes in the informatization of the production process, in particular, permanent structures of temporary dentures that are necessary in the structure of dental orthopedic treatment of patients with partial absence of teeth [4]. The introduction of integrated management systems and economic and mathematical models makes it possible to optimize the main business indicators of a dental organization and justify current capital costs, predict the effectiveness of its activities, evaluate any activity in the perspective of long-term development goals, and also provide reliable statistics [5].

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Digitalization allows for optimization of the process of managing the life cycle of designing dentures, effective control of the quality of the production protocol, reduction of the number of frequent violations associated with the human factor [6], provision of effective dental orthopedic treatment of the dentofacial system [7], and prevention of caries in abutment teeth [8].

Based on the above, it can be assumed that an economic justification for the need for the components of the final product from the idea of its creation to practical implementation and delivery to the clinic will make it possible to determine the profitability of digital transformation, and the study of the variables of a multi-cycle process will contribute to the reliability of the forecast for the sustainability of the dental business.

The main contradiction in the process of managing the provision of dental services is that the existing scientific and methodological support for the processes of managing the development of dental medical organizations does not create the prerequisites for increasing the efficiency of their work, improving the quality and availability of dental care to the population, introducing advanced technologies for material incentives for dental and administrative and managerial personnel, advanced technologies for financing dental medical organizations, and arranging their work with regard to the scientifically grounded mathematical apparatus for making managerial decisions.

Despite the high social significance of dental care, currently the processes of managing dental services are characterized as far from being effective since there is no holistic, scientifically grounded approach to making managerial decisions based on economic-mathematical, statistical, and instrumental methods and models integrated into everyday dental practice. Many dental medical organizations have a significant disparity in the provision of dental care in public and commercial dental organizations. There is a significant gap between the technical equipment of dental medical organizations in the public and commercial sectors and in planning and organizing dental care in different regions of the Russian Federation, which undoubtedly imposes high demands on the development and practical implementation of modern economic and mathematical models for managing dental services based on functional modeling, linear and nonlinear programming, and equations of A.N. Kolmogorov and Markov random processes that meet the demands of the population and the country's leadership. Thus, the development of models for managing dental services based on the Markov random process and Kolmogorov's equations is an essential and relevant scientific task for the national economy.

1.2. Management of Dental Services as a Markov Random Process

1.2.1. Markov Random Process

When researching operations, one often encounters systems designed for multiple uses when solving similar tasks. The processes that arise are called *service processes*, and the systems are called *queuing systems* (QS). Telephone systems, repair shops, computer complexes, ticket offices, shops, hairdressers, service provision, etc. are examples of such systems. Each QS consists of a certain number of service units (instruments, devices, points, and stations), which we will call *service channels*. Communication lines, work points, computers, salespeople, dentists, dental technicians, etc., can be channels. Based on the number of channels, queuing systems are divided into *single-channel* and *multi-channel*. Applications are usually received by the QS not regularly but randomly, forming a so-called *random flow of applications (requests).* Generally speaking, servicing of requests also continues for some random time. The random nature of the flow of requests and service time leads to the fact that the QS is unevenly loaded: in some periods a very large number of applications accumulate (they either get queued or leave the QS unserved); in other periods the QS operates with underload or is idle. *The subject of queuing theory* is the construction of mathematical models that connect the given operating conditions of the QS (the number of channels, their productivity, the nature of the flow of requests, etc.) with performance indicators of the QS, describing its ability to cope with the flow of requests.

The QS operation is a *random process*. A *random (probabilistic or stochastic*) *process* is understood as a process of changing the system state over time in accordance with probabilistic laws. A process is called a *discrete-state process* if its possible states S_1, S_2, S_3, … can be enumerated in advance, and the transition of the system from one state to another state occurs instantly (in a jump). A process is called a continuous-*time process* if the moments of possible transitions of the system from one state to another state are random rather than fixed in advance. The QS operation process is a random discrete-state and continuous-time process. This means that the state of the QS changes abruptly at random moments when some events occur (for example, the arrival of a new request, the end of service, etc.). The mathematical analysis of the QS operation is significantly simplified if the process of this operation is Markovian. A random process is called a *Markov* or *random process* without aftereffect if, for any moment of time t_0, the probabilistic characteristics of the process in the future depend only on its state at a given moment t_0 and do not depend on when and how the system came to this state. Many processes, including the provision of dental services, can be approximately considered Markov processes. In some cases, one can simply neglect the prehistory of the processes under consideration and use Markov models to study them. When analyzing random discrete-state processes, it is convenient to use a geometric scheme – the so-called *state graph*. Usually, system states are depicted by rectangles (circles), and possible transitions from one state to another state are shown by arrows (oriented arcs) connecting the states.

1.2.2. Flows of Events

A *flow of events* is understood as a sequence of homogeneous events following one after another at some random moments in time (for example, a flow of calls at a telephone exchange, a flow of computer failures, a flow of customers, a flow of patients to a dentist, etc.). The flow is characterized by *intensity* λ - the frequency of occurrence or the average number of events entering the QS per unit time. The flow of events is called *regular* if events follow one another at certain equal intervals of time. A flow of events is called *stationary* if its probabilistic characteristics do not depend on time. In particular, the intensity of a stationary flow is a constant value: $\lambda(t)= \lambda$. A flow of events is called a *flow without aftereffect* if, for any two non-overlapping time periods τ_1 and τ_2, the number of events falling on one of them does not depend on the number of events falling on the others. A flow of events is called *ordinary* if the probability of two or more events occurring in a small (elementary) time interval Δt is negligible compared to the probability of one event occurring. In other words, a flow of events is ordinary if events appear in it in singles rather than in groups. *A flow of events is called simplest* (or *stationary Poisson process*) *if it is simultaneously stationary, ordinary, and has no aftereffect*. The name "simplest" is explained by the fact that QS with the simplest flows has the simplest mathematical description. Note that a regular flow is not the "simplest" one, since it has an aftereffect: the moments of occurrence of events in such a flow are strictly fixed. The simplest flow as a limit one arises in the theory of random processes as naturally as the normal distribution in the theory of probability is obtained as a limit for the sum of random variables: *with the imposition (superposition) of a sufficiently large number n of independent, stationary, and ordinary flows (comparable to each other in intensity λ_i (i = l, 2, ..., n), the result is a flow close to the simplest one with an intensity λ equal to the sum of the intensities of the incoming flows*, i.e.

$$\lambda = \sum_{i=1}^{n} \lambda_i \tag{1}$$

1.2.3. Managing Dental Services

In managing dental services, it is a frequent practice to deal with situations of multiple repetitions of similar tasks. In its economic essence, the provision of dental care is a system of mass services, and the work of a dentist is an example of a random, probabilistic, stochastic process of changes over time in patients' conditions and the dental care system itself in accordance with probabilistic patterns.

Mathematical analysis of a queuing system operation is significantly simplified if this is a Markov process [9]. A random process is called a *Markov process* or *random process without aftereffect* if, for any moment t_0, the probabilistic characteristics of the process in the future depend only on its state at a given moment t_0 and do not depend on when and how the system came to this state. Many processes can be approximately considered Markov ones. Thus, the provision of dental services is a Markov random process, since the system state is characterized by the intensity of the flow of applications for dental treatment at time t_0, the condition of the patients' oral cavity, their dental status, and the ability of the system to service the incoming flow of applications, considering the satisfaction of patients' requests for availability and quality of service provision. The probability that at time $t > t_0$ all patients will be provided with dental services and will be cured depends primarily on the system state at a given time t_0, rather than on how, when and the sequence in which patients sought help from a dental medical organization.

In a number of cases, the background of the random processes under consideration can be neglected, and the Markov random process, Markov models, and the limiting probabilities of states and Kolmogorov equations can be used to study them [10–12]. When analyzing random processes with discrete states, it is convenient to use a geometric scheme—a *state diagram*. In practice, the system states are usually represented by rectangles (circles, ovals), and possible transitions from one state to another are represented by arrows (directed, oriented arcs) connecting the system states.

Figure 1 represents the process of providing dental care to the population in the form of such a diagram.

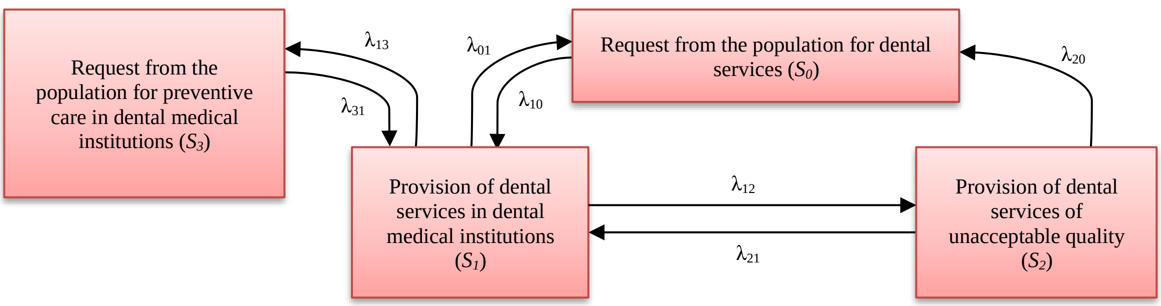


Figure 1. A diagram of possible states of the process of providing dental care to the population

State S_0 characterizes the population's request for dental services.

The majority of dental specialists, health care managers, and epidemiologists agree that at least 99% of the Russian population needs regular dental services [13–15]. Moreover, the need for dental care is distributed extremely unevenly across regions of the Russian Federation and gender and age groups. For example, according to Dakhkilgov [16], in the Republic of Ingushetia, the prevalence of caries reaches 100% for the age group of 35-44 years. Kuznetsova [17] also revealed the prevalence of caries in the age group of 35-44 years, which was 100% in all the areas she surveyed. The data obtained indicate a lack of prevention of dental diseases and low, untimely access to dental care, a widespread belief among rural residents that decayed teeth are simpler and easier to remove than to treat. Thus, the idea of dental care that is widespread and current in the Russian regions and especially in rural areas can be schematically represented as follows (see Figure 2): 1) removal of diseased or damaged teeth; 2) prosthetics; 3) treatment of caries and its complications. This means that the treatment of caries and its consequences is ranked as the last in the hierarchy of needs of rural residents of the Russian Federation, since, in the popular belief, it does not deserve the expenses, torment, and red tape that accompany the long and sometimes expensive process of treating carious cavities, and even more so the complications of caries. According to a significant contingent of patients in rural areas and remote corners of Russia, it is much simpler and involves less hassle to get rid of a diseased tooth because there will still be enough teeth in the mouth for a normal and quiet life.

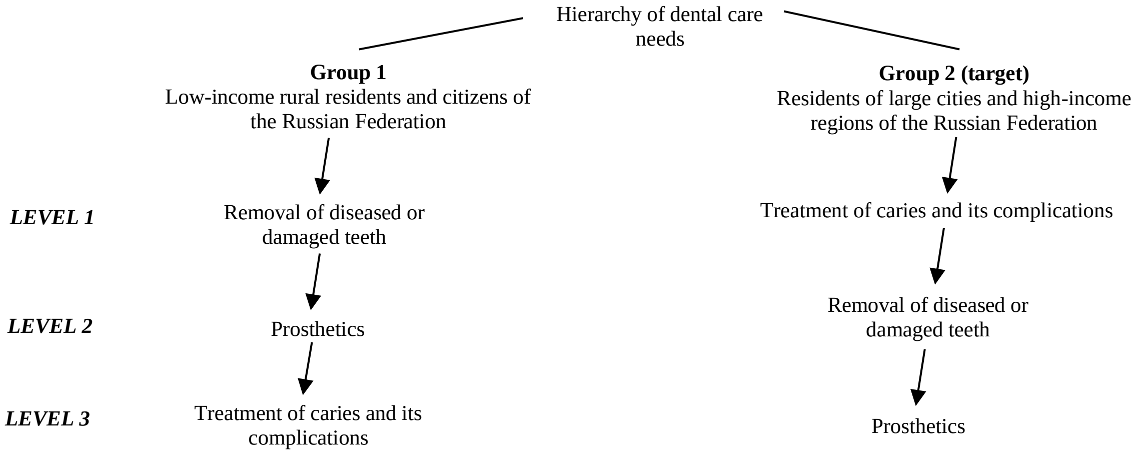


Figure 2. Schematic representation of the hierarchy of dental care needs for the population of the Russian Federation

At the same time, for large cities of the Russian Federation and regions with a high level of well-being of residents, the hierarchy of dental care needs looks different than for rural residents and the low-income population of the Russian Federation, namely: 1) treatment of caries and its complications; 2) removal of diseased or damaged teeth; 3) prosthetics.

Group 2 (see Figure 2) is the target group in this study: residents of large cities and high-income regions of the Russian Federation. Therefore, within the framework of this study, the main nosological form of dental care is the treatment of caries and its complications (the first level of needs for target group 2).

Alimsky [8] showed that 100% of elderly people aged from 60-69 years to 90 years and older are affected by dental caries. Rashkovsky [14] examined elderly and senile people and centenarians located in large clinical and gerontological hospitals in Moscow. A high level of dental caries was revealed (100%) in all age groups.

Research by Kurbanov [11] showed that the prevalence of caries among the population of the surveyed areas turned out to be quite wide. Already at the age of 12-19 years, it was 60%-70% and, increasing further, reached 98-99% in older age groups; moreover, the prevalence of dental diseases varied in different areas.

According to Abakarov [7], the prevalence of caries among patients who sought treatment and preventive care in the state dental structures of Makhachkala is quite high. Already at the age of 20-29 years, it reached 83.37%, increasing with age, reaching 96.6% in older age groups. Thus, the studies presented above give reason to believe that the population's demand for dental services (S_0) is 100%.

In Figure 1, parameter λ_{01} represents the intensity of the flow of applications from the population for dental services, understood in this case as the number of patients receiving dental services.

In the framework of this study, for simplicity and without loss of economic meaning, preventive care of the population by dental medical organizations is not considered, therefore in Figure 1 request for preventive services in dental medical organizations represents state S_3 and is not examined in this research.

Let us consider the provision of dental care by one of the dental medical organizations in Moscow for the period from September 2022 to August 2023 (one calendar year) as an example.

According to the statistical collection "Healthcare in Russia (2021)" [18-21], the number of medical organizations with dental departments (offices) is 3,486. According to the Federal State Statistics Service [16], the permanent population of the Russian Federation as of January 1, 2023 is estimated 146,447,400 people. Therefore, 42,010 people are served by one dental medical organization. As shown above, the need for dental services is estimated at 100%, which means that the population's demand for dental services per one dental medical organization will be estimated at 42,010 people · 100% = 42,010 people.

We will take the time period for the Markov random process of providing dental care to the population equal to one calendar year, namely from September 2022 to August 2023. Then λ_{01} = 42,010 people.

According to data provided by one of the dental medical organizations in Moscow, during the period under review, 1,874 dental services were provided, which means λ_{01} = 1,874 services (see Figure 1).

- State S_1 denotes provision of dental services in dental medical organizations. At this stage of the Markov random process, the patient has two possible transitions from state S_1: to state S_0 with intensity λ_{10} or to state S_2 with intensity λ_{12}.
- State S_2 is characterized by the proportion of dental services provided that are of unacceptable quality for the patient. The outcome of such a state for the patient can be either a return to state S_1 – repeated provision of dental services in the same dental medical organization to eliminate the defects that have arisen, or a transition to state S_0, when the patients refuse to continue dental treatment in the dental clinic where they received poor-quality service, and replenish the pool of patients forming a request from the population for dental services.

According to surveys of patient reviews on the quality of dental services provided in social networks, official websites of dental medical organizations and other open sources, Table 1 shows the average shares of dental services provided, which, in the opinion of patients, do not meet quality criteria and need to be corrected, or return visit to a dentist. Table 1 presents the subjective opinions of patients. Evaluations of dentists, and methods of objective control over the quality of dental services provided are not shown in Table 1 as they are beyond the scope of this study.

Table 1. Average shares of dental services provided, which do not meet quality criteria in the opinion of patients

No.	Dental medical organization and official website	Average percent of substandard services
1	Mendeleyev https://me-dent.ru	4.36%
2	Academstom	4.80%
3	Stars Dental https://starsdent.ru	3.30%
4	Very Important Person	3.00%
5	Dentum	5.00%
6	Dentist	5.00%
7	Implant master	5.00%
8	Bionic Dentist	5.00%
9	Implant-center	5.00%
10	Lik	4.70%
11	Roott Dentistry https://dentalroott.ru	2.76%
12	Ilatan	4.60%
13	Dental World	4.30%
14	Medic Stom	4.70%
15	Doctor Osipova's Clinic https://doctu.ru/msk/clinic/ludent/reviews	3.35%
16	Master class	3.50%
17	Healthy Smile https://zsmile.ru	4.50%
18	Dental clinic No. 50	3.00%
19	Dental clinic No. 7 https://www.gauzsp7.ru	3.10%
20	Dentist 11	3.00%
21	Institute of Health	3.50%
22	First Social Dental Polyclinic	3.30%
23	Dentistry No.1	3.00%
24	Dentistry No. 4	4.50%

25	Moscow Dentistry No. 1	4.00%
26	North-Eastern Dental Center No. 2	3.00%
27	Nature Therapy Research Institute https://www.naturmed.ru/	3.00%
28	Dental Center of the I.M. Sechenov First Moscow State Medical University	2.50%
29	Dental Polyclinic No. 64	1.50%
30	IQ Denta Dentistry https://iqdenta.ru	6.00%
31	MEDEST Dental clinic https://medest.ru	6.50%
32	KDC Clinic https://kdcclinic.ru	2.60%
33	Gnathology, Orthodontia and Prosthetics Clinic https://my-ort.ru	6.00%
34	32dent Dentistry https://www.32dent.ru	2.00%
35	Prezi-Dent Dental Complex https://www.prezi-dent.ru	2.15%
36	Caries.net Dental Network https://kariesy.net	4.00%
37	Aristocrat-Dent Dentistry https//aristocratdent.ru	5.13%
38	22nd Century Dentistry https://www.22clinic.ru	3.40%
39	Light Dentistry https://legstom.ru/otzyvy	5.67%
40	Dental Max Private Dental Clinic https://www.dental-max.ru/29-farforovye-viniry.html	5.00%
41	Vimontale https://vimontale.ru/uslugi/protezirovanie-zubov/zubnye-mosti	9.00%
42	Our Dentist https://nashdantist.ru/keramicheskij-most	5.00%
43	Dentistry of Amazing Prices https://chudostom.ru/articles/zubnoj-most-iz-metallokeramiki	2.00%
44	Doctor Levin https://doctorlevin.ru/most-na-zuby	7.00%
45	Samson-Denta https://samson-denta.ru/services/protezirovanie-zubov/mostovidnie	4.00%
46	Dentoprofile https://www.dentoprofile.ru	5.00%
47	Rio-Stom https://rio-stom.ru	3.00%
48	My Ortodont https://myortodont.com/?utm_source=gmb	6.00%
49	Beltanova https://beltanova.ru	2.00%
50	Lanri Clinic https://lanri-klinik-med.obiz.ru/?utm_source=gmb	3.00%
51	Bazis-Med https://bazis-med-med.ru/?utm_source=gmb	4.00%
52	High Guard https://hgclinic.ru/?utm_source=gmb	7.00%
53	Linar Dent https://linardent.ru	3.00%
54	Dental https://32-zuba.com	4.00%
55	Dentatech https://dentatech.ru	7.00%
56	Stoma Fresh https://stomafresh.ru	6.00%
57	Atlantis https://atlantis-dental.ru/?utm_source=google-map	2.00%
58	"Esthetics" Dental Centers https://estetikx.ru	4.00%
59	Esthetic Classic Dent https://esthetic-classic-dent-stom.obiz.ru/?utm_source=gmb	6.00%
60	Aesthetic A centery https://aestclinic.ru	3.00%
61	Dental Art https://dentalart.ru	2.00%
62	Happy Dents https://happydents.ru	3.00%
63	Dental Health Center https://dhcentre.ru/contacts	5.00%
64	Inwhite Medical https://inwhite-medical.ru	2.00%
65	"Implant Clinic" Dental Clinic https://implantclinic.ru	3.10%
66	Izub Dental Center https://izub.ru	4.10%
67	Stomdom Dental Clinic https://stomdom.com	3.50%
68	Dentistry in Serpukhovskaya https://stomatology-center.ru	3.45%
69	Zublandia Dental Clinic https://zublandia.ru	2.70%
70	Vse-Svoi Dental clinic https://vse-svoi.ru	2.95%
71	Dantistoff Scientific Dentistry https://dantistoff.ru	1.73%
72	Zub.ru Dental Clinic https://zub.ru	1.52%
73	"Doctor Smile" Dental clinic https://doctorSmile.ru	6.50%
74	"Beauty Smile" Dental clinic https://beautySmile.ru	5.87%

75	"Astra Dent" Dental clinic https://astradentstom.ru	4.35%
76	"Dental Butik" Dental clinic https://dentalbutik.ru	1.30%
77	"Clinic ln" Dental clinic https://clinicln.ru	0.50%
78	"Dental Fantasy" Dental clinic https://dentalFantasy.ru	3.00%
79	"SLclinic" Dental clinic https://slclinic.ru	1.60%
80	"AO-Stom" Dental clinic https://ao-stom.ru	2.40%
81	Roden Dental clinic https://rodendent.ru	5.62%
82	"Doctor Jean" Dental clinic https://drjean.ru	2.00%
83	"Specialist" Dental clinic https://dentSpecialist.ru	7.00%
84	"Canadian Dentistry" Dental clinic https://canadaDent.ru	4.00%
85	"Archident" Dentistry in Konkovo https://doctu.ru/msk/clinic/archident/reviews	0.55%
86	"Rich-Dent" Dentistry https://richdent.ru	2.75%
87	Ru-Denta https://msk.stom-firms.ru/clinics.php?i=2750	2.75%
88	Unident https://msk.stom-firms.ru/clinics.php?i=1930	2.90%
89	NAVA dental clinic https://nava.ru	1.10%
90	Implant City https://implantcity.ru	3.77%
91	Diamond Dent https://msk.stom-firms.ru/clinics.php?i=3107	2.25%
92	Dental polyclinic No. 15 https://sp15.mos.ru/?ysclid=115ipn53y7	2.00%
93	"Denta-Ehl" in Polezhaevskaya https://doctu.ru/msk/clinic/denta-ehl-na-polezhaevskojj/reviews	2.35%
94	"Denta-Ehl" in Sevastopolskaya https://doctu.ru/msk/clinic/dentol-l/reviews	3.50%
95	"Denta-Ehl" in Ilyicha Square https://doctu.ru/msk/clinic/denta-ehl-na-pl-ilicha/reviews	1.75%
96	"Denta-Ehl" at the University https://doctu.ru/msk/clinic/denta-ehl-na-universitete/reviews	2,80%
97	"Denta-Ehl" in Skhodnenskaya https://doctu.ru/msk/clinic/denta-ehl-na-skhodnenskojj/reviews	3.05%
98	"Denta-Ehl" in Ushakova https://doctu.ru/msk/clinic/denta-ehl-na-ushakova/reviews	1.90%
99	"Archident" Dentistry in Mitino https://doctu.ru/msk/clinic/archident-2/reviews	2.80%
100	"Jumi" Dentistry https://doctu.ru/msk/clinic/stomatologija-jumi/reviews	1.10%
101	"ABC Medicine" in the Park of Culture https://doctu.ru/msk/clinic/abc-medicina-parkcultura/reviews	0.35%
102	"Stolitsa" Clinic Network https://stomed.ru	3.20%
103	"Sesil Plus" Clinic https://doctu.ru/msk/clinic/stomed-arbat/reviews	3.50%
104	"Health" Clinic in Varshavskaya https://doctu.ru/msk/clinic/medcentrbiz/reviews	4.00%
105	"King Smile" Dentistry https://doctu.ru/msk/clinic/king-smile/reviews	2.03%
106	"Medservice Beauty" Dental clinic https://doctu.ru/msk/clinic/medbeauty/reviews	3.35%
107	SM-Dentistry https://www.smclinic.ru/clinics	6.50%
108	"Denta Vita" Dentistry in Novokuznetskaya https://doctu.ru/msk/clinic/dentavita-1/reviews	2.75%
109	"Denta Vita" Dentistry in the Park of Culture https://doctu.ru/msk/clinic/dentavita-1/reviews	2.10%
110	Dentistry of Affordable Prices https://msk.stom-firms.ru/clinics.php?i=10104	1.00%
111	"Novadent" Dentistry https://novadent.ru	2.50%
112	"Implantology Center" Dentistry https://impl.ru	1.00%
113	"MEDEST" Dental clinic https://medestlub.ru	6.00%
114	Veronika Dentistry https://tavi-dent.ru	4.00%
115	"Raufberg" Dentistry https://raufberg.ru/uslugi	5.00%
116	Dental polyclinic No. 60 https://sp60.mos.ru	8.00%
117	Dental polyclinic No. 32 https://sp32.mos.ru	5.00%
118	Dental polyclinic No. 67 https://сп67.рф	4.00%
119	Dental polyclinic No. 24 https://mosstom24.ru	3.00%
120	Dental polyclinic No. 66 https://сп66дзм.москва	3.00%
121	Dental polyclinic No. 62 https://sp62.mos.ru	7.00%
122	Dental polyclinic No. 65 https://sp65.mos.ru	5.00%
123	Dental polyclinic No. 5 https://stompol5.ru	6.00%
124	Dental polyclinic No. 11 https://sp11.ru	3.00%

125	"Espadent" Dentistry https://www.espadent.ru	7.00%
126	"Shifa" Dentistry https://shifa-msk.ru	10.00%
127	"Identi" Dentistry "Иденти" https://identi-clinic.ru	4.00%
128	"Polyclinic Otradnoye" Dental clinic https://polyclin.ru	5.00%
129	"Impla Dent" Dentistry https://impladent.ru	1.00%
130	"Dental Guru" Dentistry https://dentalgu.ru	4.00%
131	"ViDentist" Dentistry "" https://videntis.ru	6.00%
132	"Wisdom Tooth" Dentistry https://gavrilovdent-med.ru	6.00%
133	"Family Dentistry" https://smile4you.ru	4.00%
134	"SmileLike" Dentistry https://smilelike.ru	5.50%
135	"Doctor Dynasty" Dentistry https://dinvrach.ru	7.00%
136	"Colibri Dental" Dentistry https://colibrident.ru	9.00%
137	"Smile Spa" Dentistry https://smilespa.ru/ceny-na-stomatologiyu.html	6.00%
138	"DentoSpas" Dentistry "ДентоСпас" https://dentospas.ru	4.00%
139	"Capital Smile" Dentistry https://capitalsmile.ru	3.00%
140	"ProfiStyle" Dentistry https://profistyle.moscow	8.00%
141	"Vitam" Dentistry https://medvitam.ru	3.00%
142	"Art Esthetics" Dentistry https://estetic-art.ru	8.00%
143	"Dr. Kogina" Dentistry https://drkogina.ru	2.00%
144	"RocosClinic" Dentistry https://rocosclinic.ru	6.00%
145	"Kalinin Dentistry" https://kalinindentistry.ru	4.00%
146	"Lucky Smile" Dentistry https://ls-stom.ru	7.00%
147	"Medical Star" Dentistry https://medical-star.ru	3.00%
148	"WestMed" Dentistry https://westmed-st.ru	5.00%
149	Partner-Med Dentistry https://partner-med.com	8.00%
150	Oriondent Dentistry https://oriondent.ru	4.00%
151	"Smile at once" Dentistry https://smile-at-once.ru	2.00%
	Mean Value	3.97%
	Mean-Square Deviation	1.90%

Thus, according to the data presented in Table 1, 3.97% of dental services provided do not meet quality criteria; therefore, λ_{12} = 0.0397 (share of substandard dental services) 1,874 services = 74 services per year. It should be noted that after a patient has been provided with a substandard dental service, some patients agree to have it corrected in the same dental medical organization where the poor-quality service was provided, while other patients correct the dentists' mistakes in other dental clinics or refuse from further treatment. Let us denote through the parameter $0 \leq \varphi \leq 1$ the proportion of patients who agree to correct defects in the same dental medical organization where the substandard service was provided. And we represent the proportion of patients who correct the dentists' mistakes in other dental medical organizations profile or refuse further treatment through $1 - \varphi$.

This means, according to Figure 1, that $\lambda_{21} = 74\varphi$, a $\lambda_{20} = 74 \cdot (1 - \varphi)$.

1. State S_3 represents a request from the population for preventive services in dental medical organizations. It is characterized by the intensity of flows of requests for preventive services from the population (λ_{31}) and flows of preventive services in dental medical organizations (λ_{13}). As stated above, for simplicity and without loss of economic meaning, we do not consider state S_3 in this study.

Thus, the purpose of this study is to develop a mathematical model for managing dental services as a Markov random process on the basis of the state graph of the process of providing dental care to the population, and provide its practical implementation as exemplified by a dental medical organization in Moscow in the MathCad software environment.

Research hypothesis: The development of a mathematical model of a Markov random process based on the state graph of the process of providing dental care to the population and its practical implementation in the MathCad software environment make it possible to optimize the process of managing dental services.

2. Literature Review

According to the United Nations forecasts, the world's population will grow to 8.5 billion by 2030, to 9.7 billion by 2050, and to 10.9 billion by 2100. Along with this demographic collapse, life expectancy continues to rise especially in countries with a high level of prosperity [22], which unconditionally creates a massive population of patients in need of dental care.

According to a number of authors, Carlsson & Omar [23] and Aputiunov et al. [24] complete tooth loss identified among the elderly and senile population in different regions of Russia amounted to 31.5–41.2% of cases. This pathology was more common in elderly and senile men and women in rural areas than in urban residents. According to WHO, up to 26% of people with complete loss of teeth do not use dentures for various reasons, and 37% are forced to adapt to them for a long time.

In the structure of the concept of healthy aging of the population of planet Earth, put forward by the World Health Organization, special attention is paid to "Oral Health", which is associated with conditions related to breathing, eating, communication, the ability to smile, establish social connections, fully realize one's potential and participate in public life. The listed components are determinants of healthy aging and reflect the social, physical quality of life and mental well-being of people of geriatric age, and closely depend on dental health [25, 26].

Many domestic researchers become perplexed by the issue of treating patients with complete teeth loss, especially geriatric patients, since disablement has not only functional (impaired diction, loss of the ability to speak, and difficulties associated with inadequate food intake lead to problems with the gastrointestinal tract [27], characteristic geriatric syndromes, poly[co]morbidity, polypragmasy, critical decrease in the level of quality of life), but also the esthetic side of the problem. A senile appearance is being formed. Persons with such complex pathology require comprehensive and, as a rule, interdisciplinary treatment and rehabilitation [28].

Prosthetic treatment of patients with complete teeth loss is a complex task; it cannot always be solved successfully. In some cases, polymer structures contribute to the transformation of the homeostasis of the oral cavity and the entire body as a whole; waste products of microbial associations aggravate the pathology [29, 30], which affects the quality of life of users of complete removable dentures [31, 32].

Adaptation to complete dentures is hampered, which worsens their fixation, and patients refuse to use them. According to Shurygin et al. [27], in the patients over 60 years of age they examined, repeated prosthetics were required in 38.5±7.3% of cases.

The medical and social model of preferential dental prosthetics does not allow to avoid cases of repeated and multiple visits to health care institutions due to ineffective results of prosthetics. Patients receive prostheses that do not meet the requirements of functionality, esthetics and long-term use (5 years or more), which significantly reduces their quality of life and the quality of social functioning [25, 33, 34]. In addition, these problems place a heavy economic burden on the shoulders of the state, as the number of requests from citizens regarding preferential dental prosthetics increases significantly; the quality of services decreases and does not contribute to the rational and efficient use of budget funds [35, 36].

Thus, the review of the literature devoted to the issues of managing dental services and the need for dental services of the population showed a lack of studies aimed at optimizing the management of dental services using Markov random processes. Currently, there is no single theoretical and methodological approach to creating mathematical models of dental services management processes, and the problem of their scientifically based mathematical modeling has not been completely resolved, which determines the relevance of the topic of this research. Mathematical modeling of Markov random processes in relation to the development of dental care is relevant and has scientific and practical value. As opposed to the dental service management models used in practice [28, 30, 33], this study has developed a Markov chain-based graph that allows for visual and schematical presentation of the full cycle of processes for providing dental services to the population, including the patient's request for dental care and the abilities of dental medical organizations to satisfy this ever-increasing request with the required quality of services provided. Moreover, it justifies the need for preventive maintenance of the population, which is the fundamental basis for the development of mathematical models of Markov random processes and A.N. Kolmogorov's equations and makes it possible to avoid mistakes at the stage of model design and practical implementation.

At the same time, analysis of Russian and foreign studies [37-40] specializing in the area of using Markov random processes to solve practical problems showed that, as a rule, the authors pay increased attention to the mathematical apparatus of the Markov process and its detailed study, leaving outside the scope of their attention the practical component of controlling various processes using Markov chains and graphs. Thus, in the study by Malinkovsky [34], the concept of an asymptotically Markov random process is introduced, and it is proved that for a wide class of homogeneous queuing networks, the coordinates of the state vector are asymptotically equilibrium Markov processes; however, no conclusions are drawn about where these results can be used in practice. The research by Arutunov et al. [22] analyzes existing approaches to assessing the effectiveness of methods for using aviation weapons and proposes the use of non-stationary Markov random processes to consider changes in assessing the probability of hitting objects.

The study by Huang & Chen [37] developed an innovative approach to solving the problem of independence between criteria in the hierarchy analysis method by including discrete Markov random fields in the structure of the hierarchy analysis method, which improves the process of making managerial decisions by effective and rational consideration of the interdependencies between the criteria, reflecting their actual weighting coefficients.

Thus, at present there is no universal approach to the development of scientifically grounded economic and mathematical models and mechanisms for assessing the effectiveness of the functioning of dental medical organizations based on Markov random processes, which requires the development of scientifically grounded tools adequate to the current state of the industry.

This study proposes an approach that enables optimization of the operating parameters of a dental medical organization using Markov random processes, which makes it possible to determine such a combination of operating parameters affecting the profitability of dental medical organizations, in which the economic efficiency of managerial decisions reaches its maximum value at existing restrictions on changes in the influencing factors of the model.

3. Material and Methods

3.1. A.N. Kolmogorov's Equations

Let us consider a mathematical description of a Markov random process with discrete states and continuous time as exemplified by a random process of providing dental care to the population, whose diagram is shown in Figure 1. We assume that all transitions of the system from state S_i to state S_j occur under the influence of simple streams of events with intensities λ_{ij} (i = 0,1,2; j = 0,1,2). Thus, the transition from state S_0 to state S_1 will occur under the influence of the population's need for dental care, and the reverse transition from state S_1 to state S_0 will occur under the influence of the flow of patient care by dentists. The remaining transitions between the system states S_0, S_1 and S_2 in Figure 1 are conducted similarly. Preventative care for patients (state S_3) is not the purpose of this study and therefore transitions to state S_3 are not considered in the model.

To calculate the probability of finding a dental care system for patients S in one of the above states S_0, S_1 or S_2, the use of the system of Kolmogorov's differential equations for the probabilities of states in Figure 1 proved its worth:

$$\begin{cases} p_0'(t) = \lambda_{10} \cdot p_1(t) + \lambda_{20} \cdot p_2(t) - \lambda_{01} \cdot p_0(t), \\ p_1'(t) = \lambda_{01} \cdot p_0(t) + \lambda_{21} \cdot p_2(t) - (\lambda_{10} + \lambda_{12}) \cdot p_1(t), \\ p_2'(t) = \lambda_{12} \cdot p_1(t) - (\lambda_{20} + \lambda_{21}) \cdot p_2(t), \end{cases} \tag{2}$$

where λ_{ij} denotes intensities of the simplest streams of events (i = 0,1,2; j = 0,1,2); $p_o(t)$, $p_1(t)$, $p_2(t)$ are the probabilities of finding the dental care system for patients S at time t in states S_0, S_1 and S_2, respectively.

In the system of independent equations (2) there are one less than the total number of equations. Therefore, to solve the system of equations (2), it is necessary to add one more equation characterizing the principle of probability completeness, which consists in the fact that for any moment of time t the sum of the probabilities of all states is equal to one:

$$\sum_{i=0}^{2} p_i(t) = 1, \tag{3}$$

where $p_i(t)$ is the probability that at time t the system will be in state S_i.

To solve differential equations (2), it is necessary to set the initial conditions, which in this case means the initial probabilities, or, what is the same, the probabilities of the system states at the initial time t = 0. Thus, for the system of equations (1) it is natural to assume that at the initial moment of time the system is in state S_0 under initial conditions $p_0(0) = 1$, $p_1(0) = 0$, $p_2(0) = 0$.

Kolmogorov equations make it possible to find all probabilities of states as a *function of time*. Of particular interest are the probabilities of the system $p_i(t)$ in the *limiting stationary mode* at $t \to \infty$ which are called the *limiting (final)* probabilities of states.

In the theory of random processes, it has been proven [41-45] that *if the number of system states is finite and from each of them it is possible to transfer to any other state in a finite number of steps, limiting probabilities exist.*

The limiting (final) probability of the state S_i has the following meaning: it shows the *average relative time the system remains in this state*.

Since the limiting (final) probabilities are constant, replacing in Kolmogorov equations their derivatives with zero values, we obtain a system of linear algebraic equations describing the stationary mode. A flow of events is called *stationary* if its probabilistic characteristics do not depend on time. In particular, the stationary flow intensities and the limiting (final) probabilities are constant values: $\lambda(t) = \lambda$ and $p_i(t) = p_i$. Therefore, for the patient dental care system S, depicted by the state diagram in Figure 1, according to (2), the system of algebraic equations has the following form:

$$\begin{cases} \lambda_{01} \cdot p_0 = \lambda_{10} \cdot p_1 + \lambda_{20} \cdot p_2, \\ (\lambda_{10} + \lambda_{12}) \cdot p_1 = \lambda_{01} \cdot p_0 + \lambda_{21} \cdot p_2, \\ (\lambda_{20} + \lambda_{21}) \cdot p_2 = \lambda_{12} \cdot p_1. \end{cases} \tag{4}$$

System (4) can be compiled using a marked state diagram (Figure 1), if we follow the rule: on the left side of the equations there is the limiting (final) probability of a given state p_i, multiplied by the total intensity of all flows leaving a given state, and on the right side there is the sum of the products of intensities of all flows entering the considered state S_i, on the probabilities of those states from which these flows exit.

Figure 3 presents a block diagram of a comprehensive system for optimizing the management of dental services using a Markov random process, which details the main aspects and criteria for making key managerial decisions related to the management of dental services.

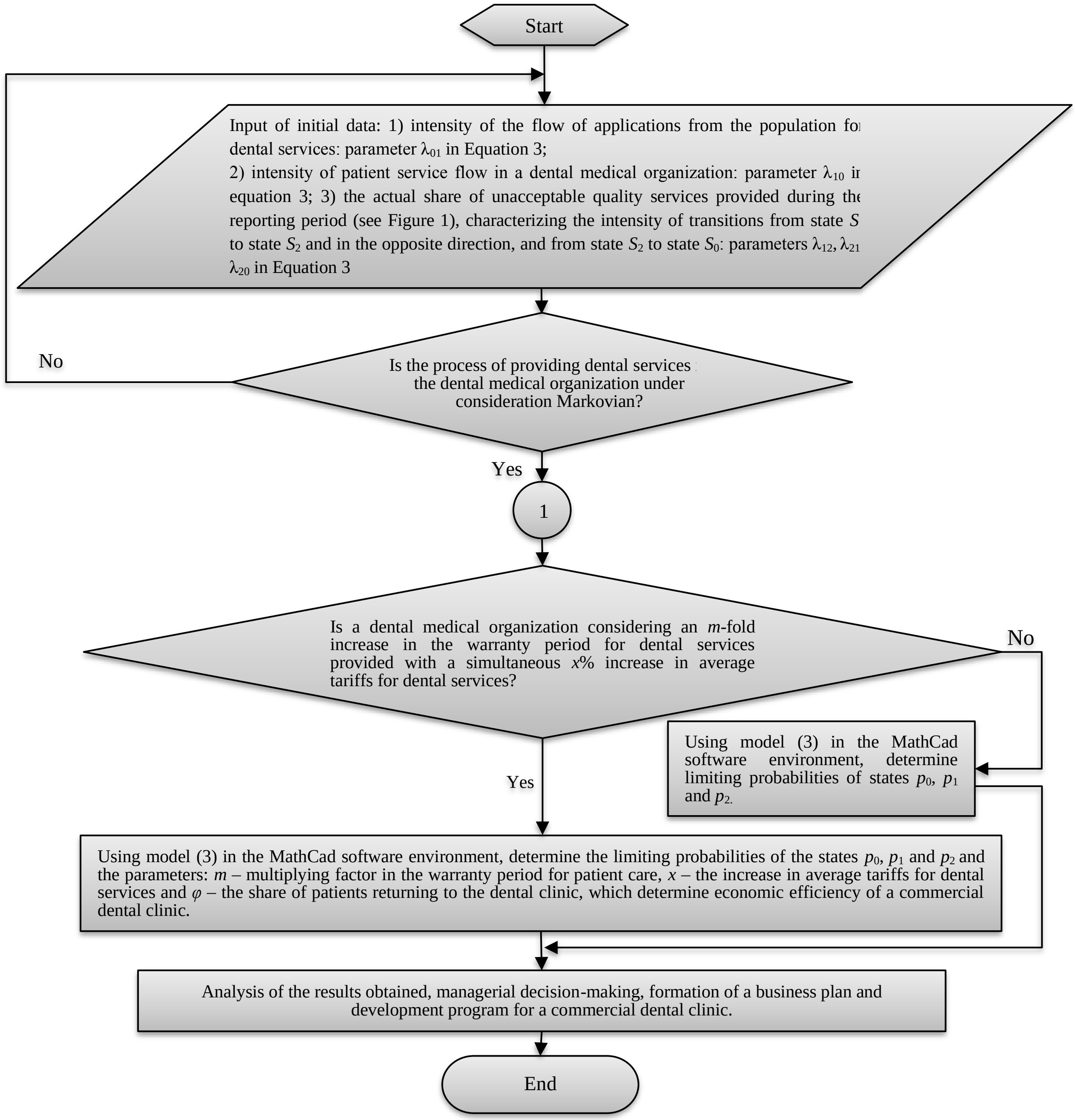


Figure 3. Block diagram of a comprehensive system for optimizing the management of dental services using a Markov random process

Comprehensive system optimization algorithm for managing dental services using a Markov random process:

Step 1. *Analysis of initial data provided by a commercial dental clinic.* At this stage, the quality, completeness and reliability of the provided information that characterizes the intensity of transitions between the states S_0, S_1 and S_2 of the Markov chain shown in Figure 1 is assessed, namely: the intensity of the flow of applications from the population for dental services: parameter λ_{01} in Equation 4; intensity of patient service flow in a dental medical organization: parameter λ_{10} in Equation 4; the actual share of services of unacceptable quality provided during the reporting period. If the data received from the top managers of a commercial dental clinic meet the required criteria, we move on to the next stage of the algorithm. Otherwise, repeat step 1 until the criteria are met.

Step 2. Checking the fulfillment of the condition that the process of providing dental services in a dental medical organization is a Markov random process. At this stage, it is necessary to test the hypothesis that the process of providing dental services in a dental medical organization is a Markov random process, or a random process without aftereffects, for which the condition is satisfied that for any moment of time t_0 the probabilistic characteristics of the process in the future depend only on its state at a given time t_0 and do not depend on when and how the system came to this state. As a rule, the process of providing dental services to the population can be approximately considered Markovian. If the condition is met, we move on to the next stage of the algorithm. Otherwise, we return to the first step of the algorithm, refine the initial parameters of the model and repeat steps 1 and 2 until the criterion of the Markov random process is satisfied.

Step 3. Checking the fulfillment of the condition whether the management of a dental medical organization is considering an m-fold increase in the warranty period for dental services provided with a simultaneous x% increase in average tariffs for dental services. If the condition is met, the parameters of the system (4) regarding the multiplying factor of the warranty period for patient care (m), the increase in average tariffs for dental services by x%, the share of low-quality dental services provided and the actual share of patients returning to the dental clinic in dental practice (φ) are set. Otherwise, the parameters of the system (4), namely: the intensity of the flow of applications from the population for dental services: parameter λ_{01} in Equation 4; intensity of patient service flow in a dental medical organization: parameter λ_{10} in Equation 4; the intensities of transitions between the system states S_0, S_1 and S_2, are set without regard to the above parameters.

Step 4. Solving the problem of managing dental services based on a Markov random process using the developed model (4). At this stage, the maximum probabilities of states p_0, p_1 and p_2 and the following parameters are determined, using model (4): m – the multiplying factor of the guaranteed period of patient care, x – the increase in average tariffs for dental services and φ – the share of patients returning to the dental clinic, which determine the economic efficiency of a commercial dental clinic. Graphs and dependencies are constructed in the MathCad software environment, enabling managerial decision-makers to determine the optimal combination of factors and model parameters (4), at which the economic efficiency of a dental medical organization will be maximum. It is also important at this stage to determine the range of changes in influencing factors, if the model is operational and the activities of a dental medical organization are effective and sustainable.

Step 5. Evaluation of the result obtained: At this stage, the results obtained are analyzed on the basis of graphs and surfaces constructed in the MathCad software environment, management decisions are made on the optimal and economically feasible increase in prices for paid dental services while simultaneously increasing the warranty period for patients; a business plan and development program for a commercial dental clinic are formulated considering the results obtained and aimed at achieving maximum financial results and economic efficiency of its activities.

Dental services management was optimized using Markov random processes in Moscow in accordance with the above flowchart presented in Figure 3 and the algorithm in the MathCad software environment.

3.2. Model Limitations

To increase the accuracy of solving the problem of the optimal combination of factors influencing the increase in financial results (gross profit) when providing dental services in dental medical organizations using Markov random processes and A.N. Kolmogorov's equations, it is recommended to consider the cost of manufacturing and selling dental services to patients, which will make it possible to develop a comprehensive economic and mathematical model for managing dental services with regard to their economic efficiency (profitability).

The economic efficiency of providing dental services to the population is significantly influenced by the structure of morbidity and the need for dental services in a region or territory, the age and gender composition of the population, the standard of living and well-being of the population and other socio-economic and environmental factors, dental equipment used in the provision of services, personnel qualifications, materials and medications, and the corporate culture and medical practice that has developed in dental medical organizations, the protocols used for the treatment and prevention of dental diseases. This study used a typical dental medical organization operating in Moscow to assess the increase in financial results (gross profit). The need for dental services of the attached population is 100%, and according to patients, the average percentage of dental services that do not meet the quality is 3. 97% with a standard deviation of 1.90% (see Table 1).

The basic premises and assumptions used in the practical implementation of dental service management models using Markov random processes include:

- The process of providing dental services is a Markov random process characterized by a simple flow of events, described in detail above in the "Introduction" section. Many processes, including the provision of dental services, can be approximately considered Markov processes.
- Patients, who act as consumers of dental services in the context of this study, act rationally, and the demand for dental services is inelastic, which allows for the management of dental medical organizations to make a managerial decision to increase the price of paid dental services while simultaneously increasing the warranty period.
- Any hidden (not explicitly stated) factors do not significantly affect the quality of the dental services provided and the patients' decision to return to the dental medical organization where the poor-quality service was provided.
- The dental medical organization is operating and will continue its economic activities in the foreseeable future.
- The dental medical organization complies with all applicable laws, regulations, patient treatment protocols, the code of medical ethics and the required level of empathy.
- A dental medical organization has, will receive or renew all necessary permits and licenses for the provision of dental services to patients issued by the relevant federal and regional authorities and organizations, on the decisions of which the results of modeling the management of dental services using Markov random processes and A.N. Kolmogorov's equations are based.

4. Results

Practical implementation of dental services management models based on the Markov random process and Kolmogorov Equations 2 to 4 is realized using the example of one of the dental medical organizations in Moscow for the period from September 2022 to August 2023 (one calendar year). It was shown above that the intensities of the simplest flows of events have the following values: λ_{01} = 42,010 people per year, λ_{10} = 1,874 services per year, λ_{12} = 74 services per year, $\lambda_{21} = 74\varphi$ and $\lambda_{20} = 74\ (1 - \varphi)$. Thus, we rewrite the system of linear algebraic Equations 3, considering the principle of probability completeness (3), in the following form:

$$\begin{cases} 42\,010 \cdot p_0 = 1\,874 \cdot p_1 + 74 \cdot (1-\varphi) \cdot p_2, \\ 1\,948 \cdot p_1 = 42\,010 \cdot p_0 + 74 \cdot \varphi \cdot p_2, \\ 74 \cdot p_2 = 74 \cdot p_1, \\ p_0 + p_1 + p_2 = 1. \end{cases} \tag{5}$$

The following values are the solution to the system of linear algebraic Equations 5:

$$p_0 = \frac{1\,948 - 74\varphi}{85\,968 - 74\varphi}; p_1 = p_2 = \frac{42\,010}{85\,968 - 74\varphi}. \tag{6}$$

Since, as stated above, $0 \leq \varphi \leq 1$, then $0.0218 \leq p_0 \leq 0.0227$, and $0.4887 \leq p_1 = p_2 \leq 0.4891$ or, which is the same, $2.18\% \leq p_0 \leq 2.27\%$, and $48.87\% \leq p_1 = p_2 \leq 48.91\%$. In other words, depending on the parameter φ (the proportion of patients who agree to correct the defect in the same dental medical organization where the substandard service was provided), the range of changes in the probability of system S being in state S_0 is only 9% (from 2.18% up to 2.27%), which may indicate that the parameter φ has virtually no effect on the population's demand for dental services (S_0).

Similarly, for the probabilities of system S being in states S_1 and S_2, the parameter φ does not have a significant effect on their values, namely: the range of changes in the probability of system S being in state S_1 or S_2 is 4% (from 48.87% to 48.91%). This means that on average, the system is in state S_1 or S_2 almost all the time, being in each of these states for the same amount of time, and only on average 2.22% of its time the system is in state S_0.

Let us find the average financial result (gross profit) for the period under consideration from September 2022 to August 2023 (one calendar year) from managing dental services, if it is known [44] that the provision of one dental service on average brings the dental medical organization in question an income of 10,567.02 rubles, and the cost of providing one dental service is 5,378.05 rubles.

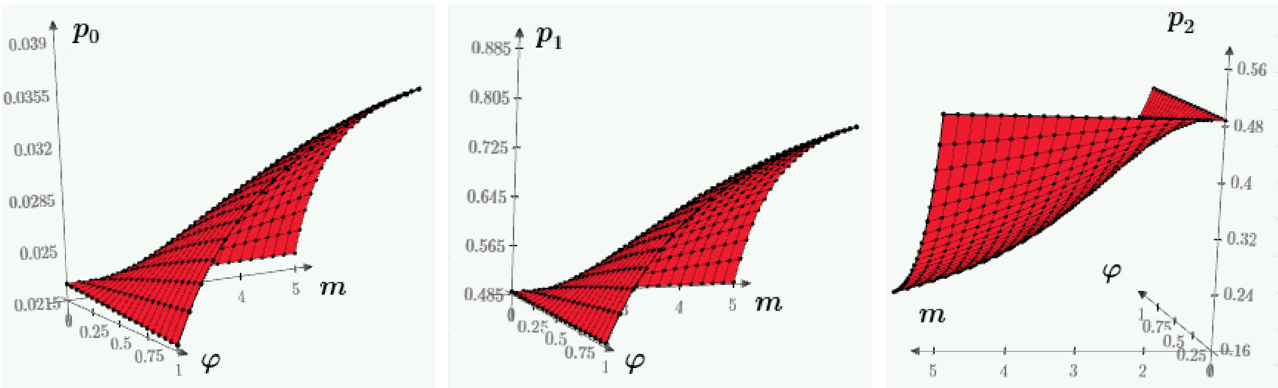
As follows from the solution to problem (5), system S is in state S_1, which is associated with dental services to the population, for on average 48.89% of its time, and the same amount of time the system is in state S_2, which is characterized by the correction of poorly provided dental services due to dental medical organization. Therefore, the average financial result (gross profit) from the provision of dental services, determined by the difference between the income from the provision of dental services and the costs of correcting poorly provided services per unit of time, is equal to 48.89% · RUB10,567.02 – 48.89% · RUB5,378.05 = RUB2,537.11.

The dental medical organization under consideration is studying the possibility of increasing the warranty period for dental services provided by a factor of m with a simultaneous x% increase in average tariffs for dental services. This means an m-fold increase in the intensity of the flow of requests for dental services to eliminate substandard dental care from 74φ services per year to 74φm services per year, i.e. in system (5) $\lambda_{21} = 74\varphi m$ services per year. And simultaneously, there is an m-fold increase in costs for correcting poorly provided services per unit of time, since, as noted above, defects are eliminated at the expense of a dental medical organization. Thus, for this option we have a system of linear algebraic equations of the following form:

$$\begin{cases} 42\,010 \cdot p_0 = 1\,874 \cdot p_1 + 74 \cdot (1-\varphi) \cdot p_2, \\ 1\,948 \cdot p_1 = 42\,010 \cdot p_0 + 74\varphi m \cdot p_2, \\ \big(74 - 74\varphi \cdot (1-m)\big) \cdot p_2 = 74 \cdot p_1, \\ p_0 + p_1 + p_2 = 1. \end{cases} \tag{7}$$

By solving it, we have: $p_0(\varphi, m) = 1 - \big(2 - \varphi \cdot (1-m)\big) \cdot p_2(\varphi, m)$; $p_1(\varphi, m) = \big(1 - \varphi \cdot (1-m)\big) \cdot p_2(\varphi, m)$; $p_2(\varphi, m) = \frac{42\,010}{85\,968 - 43\,884 \cdot \varphi \cdot (1-m) - 74 \cdot \varphi}$.

Figure 4 shows the solution to problem (7) in the MathCad software environment, provided that a dental medical organization is considering the possibility of an m-fold increase in the warranty period for dental services provided (1 year ≤ m ≤ 5 years) with a simultaneous increase of 10% in average tariffs for dental services. At the same time, the proportion of patients who agree to correct the defect in the same dental medical organization where the substandard service was provided lies in the range 0 ≤ φ ≤ 1 (the influx of new patients from other dental medical organizations is not considered).



(a) (b) (c)

Figure 4. Solution of problem (6) in the MathCad software environment

Figure 4 demonstrates that with φ = 1 and m = 5, the probability of system S being in state S_0 reaches its highest value p_0 = 0.0358 (3.58%), the probability of system S being in state S_1 also reaches its highest value p_1 = 0.8035 (80.35%), and the probability of system S being in state S_2 is minimal p_2 = 0.1607 (16.07%).

Let us study the influence of φ and m on the average financial result (gross profit), determined by the difference between income from the provision of dental services and the costs of correcting substandard services per unit of time according to the following Equation:

$$FR(\varphi, m) = RUB10{,}567.02 \cdot 1.1 \cdot p_1(\varphi, m) - RUB5{,}378.05 \cdot m \cdot p_2(\varphi, m). \tag{8}$$

We define the relative increase in the financial result (gross profit) as the ratio of the current financial result to that at the basic values of the influencing factors (initial conditions) φ = 0 and m = 1, i.e.:

$$\Delta FR(\varphi, m) = \frac{FR(\varphi, m)}{FR(0,1)}. \tag{9}$$

Figure 5 shows the dependence of the financial result (gross profit) on φ and m according to Equation 8. As follows from Figure 5 and Equation 8, the average financial result (gross profit) of the dental medical organization under consideration is maximum under the following conditions: φ = 1 and m = 5, being equal to RUB5,018.18. This means that it makes sense to increase the warranty period for dental services provided only on the condition that this will lead to an increase in the proportion of patients who agree to correct the defect in the same dental medical organization where the substandard service was provided. Otherwise, as can be seen from Figure 5, a dental medical organization will have a loss of -RUB7,460.30 at φ = 0 and m = 5. It should be noted that the threshold value is φ = 0.637. This means that with the proportion of patient returns amounting to 63.7%, the financial result (gross profit) of a dental medical organization remains unchanged with an increase in the warranty period. If the proportion of patient returns exceeds 63.7%, the financial result increases, and if it is less than 63.7%, the financial result decreases.

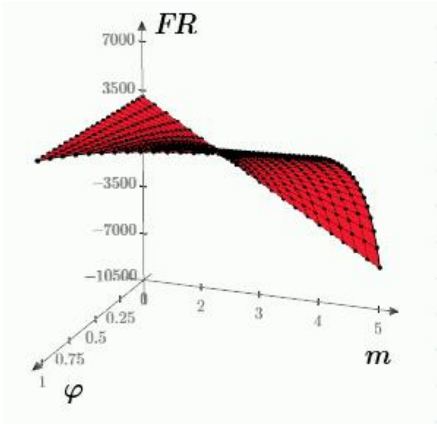


Figure 5. Impact of the proportion of patients who agree to correct the defect in the same dental medical organization where the substandard service was provided, φ, and the warranty period for the dental services provided m on the average financial result (gross profit).

Figure 6 shows the dependence of the relative average increase in financial result (gross profit) on φ and *m* according to Equation 9. As follows from Figure 6, the maximum value of the relative average increase in the financial result (gross profit) is observed at values φ = 1 and *m* = 5, amounting to 1.6442 or 64.42% compared to the base option *FR*(0,1).

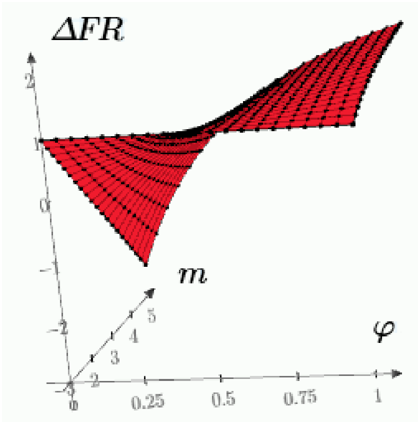


Figure 6. The influence of the proportion of patients who agree to correct the defect in the same dental medical organization where the substandard service was provided, φ, and the warranty period for the dental services provided m on the relative average increase in the financial result.

Figure 7 shows the dependence of the relative average increase in financial result Δ*FR* on the warranty period for dental services provided *m* with a full return of patients (φ = 1). As demonstrated by Figure 7, with a slight increase in the warranty period $1 \le m \le 3$, the financial result (gross profit) grows faster than with values $3 < m \le 5$. Therefore, at most three-fold increase in the warranty period of servicing is recommended for the dental medical organization under consideration, according to which Δ*FR* = 1.4851 (48.51%). To achieve a relative average increase in financial results by a factor of 1.5, the warranty period for dental services provided should be *m* = 3.13.

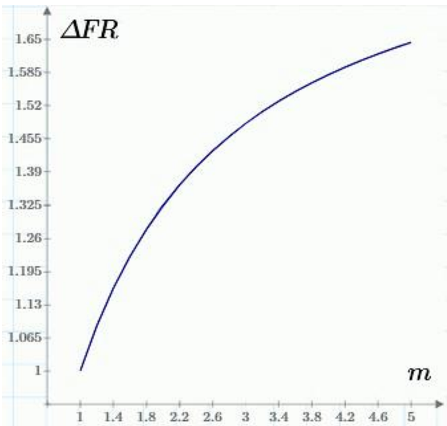


Figure 7. Dependence of the relative average increase in financial results on the warranty period for dental services provided with a full return of patients (φ = 1)

The simulation results in the MathCad software environment (see Figures 4 to 7) are presented in Table 2.

Table 2. Main results of modeling in the MathCad software environment

No.	Indicator	Indicator value	Value of parameter *m*	Value of parameter φ
1	2	3	4	5
1	Maximum value of the probability of system *S* being in state S_0, %	3.58	5	1
2	Maximum value of the probability of system *S* being in state S_1, %	80.35	5	1
3	Minimum value of the probability of system *S* being in state S_2, %	16.07	5	1
4	Maximum value of the average financial result (gross profit), RUB.	5 018.18	5	1
5	Minimum value of the average financial result (gross profit), RUB.	-7 460.30	5	0
6	Threshold value of average financial result (gross profit), RUB.	2 537.11	5	0.637
7	The maximum value of the relative average increase in financial results (gross profit), %	64.42	5	1
8	Relative average increase in financial result (gross profit), %	48.51	3	1
9	Relative average increase in financial result (gross profit), %	50.00	3,13	1

Lines 1-3 of Table 2 shows that with a full return of patients to correct the defect in the same dental medical organization where the poor-quality service was provided (φ = 1, see last column, lines 1-3 of Table 2), and the maximum warranty period $m = 5$ years (see column 4 of Table 2), the probability of system S being in state S_0 reaches its highest value $p_0 = 0.0358$ (3.58%). The probability of system S being in state S_1 also reaches its highest value $p_1 = 0.8035$ (80.35%), and the probability of system S being in state S_2 is minimal $p_2 = 0.1607$ (16.07%). These results should be interpreted as the average time the system is in the corresponding states according to Figure 1. Since this study considers a planning period of 1 year, as follows from the results obtained, with a full return of patients for correction of defects in the same dental medical organization where the poor-quality service was provided, and the maximum servicing warranty period $m = 5$ years, the system S is in the S_0 state on average 365 days a year · 0.0358 = 13.07 days; 293.28 days on average in state S_1 and approximately 58.66 days per year in state S_2.

As shown in lines 4 and 5 of Table 2, the maximum financial result from the provision of dental services in the dental medical organization in question, per dental service on average, is equal to 5,018.18 rubles, which is achieved with a full return of patients and a maximum servicing warranty period $m = 5$ years. At the same time, the minimum financial result from the provision of dental services in the dental medical organization in question, per average dental service, (maximum loss) is equal to -7,460,30 rubles, which is achieved with complete non-return of patients (φ = 0, see last column, line 5 of Table 2), i.e. when patients who have received poor-quality service correct it in another dental medical organization, and the maximum warranty period of service is $m = 5$ years.

The threshold value of the financial result per one dental service is 2,537.11 rubles (line 6 of Table 2) at φ = 0.637. This means that if more than 63.7% of patients return to a dental medical organization after providing poor-quality services for correction, in the dental organization in question the gross profit increases, otherwise the specific financial result (gross profit) decreases. In other words, the dental medical organization in question is interested in ensuring that the patient return rate approaches one.

The maximum value of the relative increase in the financial result (gross profit) with φ = 1 and $m = 5$ years is 64.42% (see column 3, line 7 of Table 2), while the main increase in the financial result (48.51%, see column 3, line 8 of Table 2) is observed with an increase in the warranty period for patient servicing $m = 3$ years, and with a further increase in the warranty period for patients up to 5 years, the specific financial result increases by only 15.91% (64.42% – 48. 51% = 15.91%). This means that it is unprofitable for a dental medical organization to increase the warranty period for patient service to five years with a full return of patients. Therefore, the value of the servicing warranty period equal to 3.13 years with a full return of patients was proposed as a threshold value in this study, at which the relative increase in the specific financial result (gross profit) will be 50% (see the last line of Table 2).

5. Analysis of the Results Obtained

- A state diagram was developed for the process of providing dental care to the population, which includes the following states: a request from the population for dental services, the provision of dental services in dental medical organizations, dental services of inadequate quality (defects) and a request from the population for preventive services.
- Based on the developed state diagram, a mathematical model for managing dental services as a Markov random process was created, which represents the limiting (final) probabilities of states of dental services, which are a particular case of the system of Kolmogorov differential equations for making managerial decisions in the theory of queuing.

- In the MathCad software environment, exemplified by one of the dental medical organizations in Moscow and the data provided by its management for the period from September 2022 to August 2023 (one calendar year), the developed models for managing dental services as a Markov random process were implemented in practice with application of limiting (final) probabilities of states and Kolmogorov equations.
- In the MathCad software environment, we studied the dependence of the probability of the system S being in states S0, S1 and S2, the financial result (gross profit) and the relative average increase in the financial result (gross profit) of the considered dental medical organization on the proportion of patients agreeing to defect correction in the same dental medical organization where the substandard service was provided (φ) and an m-fold increase in the warranty period for dental services provided (1 year ≤ m ≤ 5 years).
- It was shown (see Figure 4 and Table 2) that with a full return of patients for correction of defects in the same dental medical organization where the substandard service was provided (φ = 1) and a maximum warranty period of m = 5 years, the probability of finding the system S in state S0 reaches its highest value p0 = 0.0358 (3.58%), the probability of system S being in state S1 also reaches its highest value pi = 0.8035 (80.35%), and the probability of system S being in state S2 is minimal p2 = 0.1607 (16.07%). It was also noted (see Figures 5-7 and Table 2) that under these conditions the average financial result (gross profit) of the dental medical organization under consideration and the relative average increase in the financial result are maximum and equal to RUB5,018.18 and 64.42%, respectively.
- Calculations determined (see Figures 5 and 6 and Table 2) that the threshold value φ = 0.637. This means that with the proportion of patient returns amounting to 63.7%, the financial result (gross profit) of a dental medical organization remains unchanged with an increase in the warranty period. If the proportion of patient returns exceeds 63.7%, the financial result increases, and if it is less than 63.7%, the financial result decreases.
- Figure 7 and Table 2 shows that with a slight increase in the warranty period 1 ≤ m ≤ 3, the financial result (gross profit) grows faster than with $3 < m \leq 5$.
- Conclusions 4-7 prove the research hypothesis put forward in the Introduction.

A comparative analysis of the scientific results obtained by the authors and the results of other scientists and experts involved in certain aspects of the problems raised in this article is presented in Table. 3.

Table 3. Comparison with other studies and scientific knowledge increment

No.	Lines of research	Scientific result	Scientific novelty
1	State graph	A state graph was developed for the process of providing dental care to the population, which includes the following states: a request from the population for dental services, the provision of dental services in dental medical organizations, dental services of inadequate quality (defects) and a request from the population for preventive services	In contrast to the dental services management models used in practice [28, 30, 33] the developed graph makes it possible to visually and schematically present the full cycle of processes for providing dental services to the population, including the population's request for dental care and the ability of dental medical organizations to satisfy this ever-increasing request with the required quality of services provided, and justify the need for preventive services to the population, which acts as a fundamental basis for the development of mathematical models of Markov random processes and Kolmogorov's equations, and enables to avoid mistakes at the stage of their design and practical implementation.
2	Mathematical model, Markov random process	Based on the developed state graph, a mathematical model for managing dental services as a Markov random process was created.	In contrast to the well-known mathematical models of Markov random processes [40] this approach makes it possible to optimize the operating parameters of a dental medical organization: the warranty period for patient care, the average tariff for paid dental services and the share of patients returning to a dental medical organization who received poor-quality dental services, to correct the poorly provided service, in such a way that the activities of the medical dental organization was effective in terms of increasing financial results (gross profit).
3	Mathematical model, MathCad software environment	In the MathCad software environment, as exemplified by a dental medical organization in Moscow, the developed models for managing dental services as a Markov random process were implemented in practice using the limiting (final) probabilities of states and Kolmogorov's equations.	This approach differs from other methods for managing dental medical organizations [31, 34] by the fact that managerial decisions are made in a three-dimensional coordinate system: the multiplying factor of the warranty period for dental services provided, the share of patients returning to a dental medical organization to correct poor-quality dental services and increase the average tariff for dental services while simultaneously increasing the warranty period for them. In addition, a distinctive feature of the author's approach is that the developed mathematical model makes it possible to link the relative increase in financial results, understood as the difference between revenues from dental services provided and the costs of providing them to patients, with the main parameters of the activities of a dental medical organization and obtain dependencies that allow the management of dental organizations to make scientifically grounded managerial decisions using Markov random processes and the MathCad tool.

6. Conclusion

In compliance with the results presented in Figure 7 and Table 2, it is recommended for the dental medical organization under consideration to increase the warranty period of service by no more than a factor of 3, for which the relative average increase in the financial result will be 48.51%. To achieve a relative average increase in financial results by a factor of 1.5 times, the warranty period for dental services provided should be 3.13 years. As a result of the practical implementation of the economic and mathematical model described by Equations 2 to 4, and in accordance with the block diagram of the algorithm for introducing the Markov random process into the activities of dental medical organizations in Moscow and other regions of the Russian Federation, presented in Figure 3, this study proposes to improve the management of the provision of dental services by optimizing costs, determining the optimal balance of factors influencing the profitability of a dental medical organization and increasing the overall efficiency of the process. The authors developed a diagram of possible states of the process of providing dental care to the population, presented in Figure 1, an economic and mathematical model for managing dental services and their optimization based on a Markov random process, described by Equations 2 to 4, and block diagram of the algorithm for introducing a Markov random process into the activities of dental medical organizations in Moscow and other regions of the Russian Federation, presented in Figure 3, the basis of which is the Markov random process and A.N. Kolmogorov's equations. These developments enable managers of dental medical organizations to: a) make optimal managerial decisions in managing dental services, which will provide additional added value and reduce the costs of providing dental services during the warranty period; b) estimate the increase in financial results (gross profit) under given scenarios: the tariff for paid dental services, the period of warranty service and the share of patients returning to a dental medical organization to receive dental care under warranty service, and adjust the management of the provision of dental services by analyzing the received data using an indicator of growth in financial results (gross profit) and profitability of sales.

The diagram of possible states of the process of providing dental care to the population, presented in Figure 1, the economic and mathematical model for managing dental services and their optimization based on a Markov random process, described by by Equations 2 to 4, and the block diagram of the algorithm for introducing a Markov random process into the activities of dental medical organizations in Moscow and other regions of the Russian Federation, presented in Figure 3, which is based on the Markov random process and the equations of A.N. Kolmogorov can be used to increase the accuracy, efficiency and validity of managerial decisions in the interests of the development of dental medical organizations, increasing the profitability of their activities, improving logistics, motivating dentists to highly efficient and highly productive work, increasing its quality and modernizing dental equipment. The results of the development of scientific and methodological apparatus and the implementation of practical tools for this study allow for the conclusion that the stated goal of the study was achieved. The completed research provides managerial decision makers with effective tools for determining economic efficiency, the increase in financial results (gross profit) of dental services management using Markov random processes and A.N. Kolmogorov's equations.

Directions for further research on the studied issues include concomitant, combined, and integrated application of the developed models, a graph of possible process states in providing dental care to the population with existing models of material and moral incentives for the work of dentists and non-medical personnel to involve the entire workforce in the process of making effective managerial solutions and improving the quality of dental services provided; inclusion of the developed economic and mathematical tools into a unified information and analytical system for managing dental services, its interaction with widely used application software products, which will improve the operational management of the provision of dental services, reduce costs and increase the efficiency of business processes in the industry.

7. Declarations

7.1. Author Contributions

Conceptualization, E.V.K. and S.N.P.; methodology, G.G.B.; software, S.A.A.; validation, E.V.K, S.N.P., and G.G.B.; formal analysis, S.A.A.; investigation, G.G.B.; resources, G.G.B.; data curation, G.G.B.; writing—original draft preparation, E.V.K.; writing—review and editing, E.V.K.; visualization, S.A.A.; supervision, S.N.P.; project administration, G.G.B.; funding acquisition, G.G.B. All authors have read and agreed to the published version of the manuscript.

7.2. Data Availability Statement

The data presented in this study are available in this article.

7.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

7.4. Institutional Review Board Statement

Not applicable.

7.5. Informed Consent Statement

Not applicable.

7.6. Declaration of Competing Interest

The authors declare that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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