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# Investigating Statistical Features of the FX Bid Ask Series in a Small Economy with a Sizeable Informal Economy

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**Abstract**

The assessment of the effect of the informal use of a foreign currency on the corresponding FX rates toward the national currency is a very difficult task, requiring direct calculation and modeling. It comes mostly because of the unknown quantity of foreign money used in the informal sector, but also because of the lack of quantitative calculation models. To overcome this gap and to realize a qualitative description of this effect in a concrete economic environment, we propose herein a comparative analysis between the behavior of two typical FX rate series recorded in the Albanian currency market, the Euro-ALL and USD-ALL, providing that the Euro is used commonly as a national currency substitute in the informal economy and the USD is not. So, we have evidenced that the un-stationarity degree of the Bid and Ask spread distribution for the Euro-ALL FX series is higher than for the corresponding USD-ALL case, but with a lower variance. Those features occurring simultaneously can be explained by assuming that informal use of the Euro acts as an additional perturbation on the FX system, imposing high nonstationary, but at the same time it provides reservoir or source features for the money disbalances, reducing the average fluctuations. Next, the depth of the market measured by the average Bid-Ask Spread has resulted in a smaller price for the Euro currency, indicating a lower cost of the transactions and reinforcing the assumption regarding the distribution's in-stationarity features. Based on those indicatory findings, we propose to realize indirect evidence for our assumptions by comparing the reproduction of the corresponding distribution using autoregressive models. In this case, we have evidenced that the distribution of the FX Euro-ALL spread can be reproduced better if we include in standard ARCHX (m, n, p) models a term that mimics the informality measure. When applying the same procedure for the USD-ALL spread, the resulting distribution has not matched equally well with the original ones. Those findings have been discussed in the framework of an alternative description of the effect of the informal use of foreign currency in a small economy with a sizeable informal sector, which convene our current system under analysis, but we believe that they can be applicable for similar economic environments.

*Keywords:* Exchange Rates; q-Gaussian; Autoregressive Model; Informal Economy.

## 1. Introduction

The FX rates are the result of interactions between several endogenous economic variables, such as the balance of payments, purchasing power, prices' index, interest rates, productivity measures, import-export balances, etc., and exogenous disturbances mostly related to money supply [1, 2]. Concrete relationships between those variables are

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given in detailed econometric models shown in standard econometric literature or dedicated works as in [3-5], etc., which are based on the typical behavior of variables that are presumed measurable and known. Every disturbance or fluctuation of those cause-factors would be mirrored on the spot value of FX rates, and in principle, the trend of exchange rates is predictable and manageable theoretically. It followed that the unpredicted behavior of the FX rate is a direct result of the corresponding fluctuation of cause-factors, and therefore, if an economic environment contains additional variables or the standard ones behave specifically, the dynamics of the FX rates are expected to differ from theoretical predictions. A common factor in this category could be the alteration of the money supply by the effect of the informal economy and the use of foreign currency in the informal market in small economies [6]. From a general econometric point of view, the difference between the currency's supply and corresponding market demand represents the most characteristic features of the market. It provides guidance on market design and a means of identifying the fairness of the rents being extracted by market makers [7]. Therefore, we expect that by performing a descriptive analysis of the Bid-Ask series and their spread, we can catch the mutual influences of specific market features and FX rates for the given currency, assuming that a hidden variable is active in the system.

To explain the idea, let's consider a general perspective of the expected effect of the informal use of foreign currency on the FX rate dynamics. Consider first that in currency trading, the Bid price represents the value that forex investors are willing to pay for a (foreign) currency, and the Ask price represents the value that traders are willing to sell that currency. The Bid-Ask's magnitude, known as Bid-Ask spread, represents the depth of the currency market. Also, it is related to the underlying exchange rate uncertainty [8]. Therefore, different behavior between two Bid-Ask spread series would indicate different uncertainty levels for corresponding exchange rates. Starting from those paradigms we have proposed to compare statistical behavior between Bid-Ask spreads of the two specific FX rates to explore for specific cause that inflicted them. We expect that the informal economy and the use of foreign currency in the informal market would affect FX rates in a complex way, mostly in their dynamical parameters as discussed in Bollen et al. [7] and Prenga et al. [9], but also, those effects would redesignate the FX uncertainty, the currency's transaction costs, and market depth, which can be mirrored in the statistical properties of the corresponding spread distribution. In this regard, the goal of the work is twice as important. We want to advance in the investigation of the real effects of the informal economy in the FX rate on a certain country, Albania, and to provide some explanation on some contradictory findings of the stretch analysis about the strength and stability of national currency in conditions of the fragility of the country's' economy. Second, we want to carry out a descriptive analysis of the effect of the informal economy on the FX rates and their mutual relationships. Therefore, for this analysis, we have considered the exchange rates between the Albanian national currency (Lek-ALL) toward Euro and USD. The country's economy is too small compared with the two trading economies considered (EU and USA), so the relevant factors in the FX rates are of a domestic nature. Also, the informal economy is high, and the use of the euro in this sector is common [7, 9].

The informal economy of the country ranges in- between 0.32-0.42 fraction of the GDP [7, 10, 11]. Secondly, based on official data provided in [12–14], the inflows from remittances and the tourism industry have historically been around 9–10% of the GDP on average for several years, and by considering also the foreign currency inputs related to the real estate market, which has grown progressively last year, the net inputs in foreign currencies are estimated to be more than 20%–25% of the GDP in Bollen et al. [7]. Notice that a significant fraction of those volumes, which are known officially, can be found in the informal part of the economy. Basically, those inflows directly affect the money supply terms, which appear in equilibrium and dynamic Equations 1 and 2, exerting pressure on the trend of the exchange rates of foreign currency – domestic currency in favor of this last. This effect can compensate for a part of the disbalances in imports and exports, or even overcome them, that result in a relatively strong strengthening of the national currency, regardless of the fragile economy. Recent works on the long-term behavior of FX rates have underlined the atypical stability of the internal market values of national currencies [15, 16]. Another specific factor in the environment of FX rate formation is the position of the Central Bank and state agencies regarding the exchange rate boundaries. So, as per the Monetary Policy document approved in 2015, the value of the Albanian lek against foreign currencies is freely determined in the foreign exchange market, but the central bank could intervene to keep macro parameters within its target objective [13, 14]. We highlight the fact that, based on those policies, there is always a green light for bank measures or countermeasures that have an exterior effect regarding a currency market.

It is materialized by the daily 'open' value of FX rates. We should also underline that, after a potentially abrupt intervention, the exchange rates are accommodated by the market, which results in free marketing during trading time of the day. Notice also that there is no functional financial market in the country; therefore, most financial activities are embodied or concluded on the currency market. Bollen et al. [7] argued that due to the sizeable inward flows of foreign currency, the trend of the average FX rate changes smoothly following a Dynamical TZ curve, and the informal use of foreign currency imposes high heterogeneity at its boundaries. Considering that Bid and Ask values lie closer to those boundaries, those claims can be verified in more detailed terms. In this case, the variances of the spread would depend on the transaction costs for each currency considered, and this last can be lower for that one that is used parallelly as payment means without being converted. Specifically for the current economy, the euro is used informally as a national currency substitute, whereas the USD is mostly not, so a difference is expected regarding the deviance of their

Bid-Ask spread. Next, the open FX values are by nature close to one of the target zones, so those are highly perturbed series, whereas the closing values would mirror complex interactions between several factors that produce a better picture of the market.

Note that, as per the Bank of Albania regulation, the Bid Ask values are calculated by interbank rates during 11.30–12.00, so we missed the information about intraday changes in the FX values, implicating a further limitation of analysis. However, when considering the system as complex, we might assume that regardless of the numerous processes and hidden or expressed interactions, the heterogeneity and un-stationarity measures, etc., can be analyzed as macro parameters [7], and dissimilarities observed between observables can be identified directly. By this description and following general arguments for financial time series elaborated by Di Matteo [17], we admit that statistical features of Bid and Ask FX rates for two foreign currencies considered herein can be distinguished by their use in the informal sector. From a retrograde view, if dissimilarities are observed in the FX behavior and their Bid-Ask spread, the effect of the informal use of a currency is considered presumably known and even measurable. Finally, to avoid disturbances inflicted by a sequence of economic difficulties inflicted by the earthquake of 2019, the COVID-19 lockdown, and the Ukrainian war, we seek an apparently homogenous regime free of abnormal economic and ominous financial events, which is chosen herein from June 2021 to October 2023.

## 2. Envisaging the Effect of Informal Use of the Foreign Currency in the FX Market

Now, to explore the effect of the informal economy on the Bid-Ask spread of FX rates, let's evoke some general elements of FX value formation. Based on standard FX literature [1, 2], the equilibrium exchange rate of foreign currency with domestic currency can be expressed by:

$$FX_{rate} = \mathrm{a}_0\left(m_t - m_t^f\right) - a_1\left(y_t - y_t^f\right) - a_2\left(i_t - i_t^f\right) + \mathrm{F}\left(p_{home}^{T,NT}, p_{foreing}^{NT}\right) + \varepsilon_t \quad (1)$$

where indexes 'T', 'NT' denote tradable/non tradable, 'f' denotes 'foreign' $m_t$ are the currencies money supplies $i_t$ denote the interest rates in each economy, $y_t$are GDPs measure, $p_t$ are prices' levels (in logarithmic form), $a_i$ are constants, and $\varepsilon_t$ is a stochastic term of a Wiener type. F is a liner function. Based on this stationary relationship, disturbances in money supplies would be transferred to FX values, and clearly, informal use of foreign currency in direct transactions is a source of remarkable disturbances or theory alterations [7, 9]. Moreover, the informal economy affects directly and indirectly all economic indicators [18–20], and the country has a sizeable economy [9–11]. It followed that the expected effect on the Bid-Ask spread could be measurable for our system under scrutiny. Additional support for the effect of money disturbances and price shifting under informality pressure on FX rates and their Bid-Ask spread can be recognized by considering the dynamic FX model, for example, the Dornbusch overshooting model [21]. To highlight the idea, this model can be simplified in the following form:

$$\frac{d}{dt}\begin{bmatrix} e \\ p \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e \\ p \end{bmatrix} + \begin{bmatrix} c_1 y + c_2 m + c_3 \\ c_4 m + c_5 y - i \end{bmatrix} \quad (2)$$

where the variables (e) and (p) represent FX rate and prices in logarithm form, $A = [a, b, c, d]$ is the characteristic matrix, y is an income measure as GDP and 'i' are interest rates. Informal economy would directly modify the money supply m in (2), but the effect of the use of foreign currency in direct transaction is more complex. The variables in (2) involve more explicit terms,

$$m_t^{foreingCurrency} = m_t^{f,formal} + m_t^{f,informal};\ p_t = \begin{cases} p_t^{formal} \\ p_t^{informal} \end{cases} \quad (3)$$

and the real income measure y in (2) is unknown due to the informal part [18, 19]. Moreover, inflation measures that affect both the exchange rates and prices levels are modified in the presence of the informal economy [22]. Those elements related to the sizeable informal economy impose high uncertainty for quantitative assessment based on standard models. The situation becomes even more complicated if the foreign currency is used in the role of the national currency in direct transactions. Additionally, the Equation 2 has a saddle point solution $(e^*, p^*)$ for economic-financial circumstances where centrifugal effects inflicted by inflation pressure are present, which are common [5, 23, 24]. It followed that depending on the money supply disturbances for a given foreign currency, the dynamics of nominal exchange rates (e) would be completely different. It became clearer if we considered relations (3) for quantities involved in Equation 2. Both the saddle and nullclines would be shifted, producing distinguishable dynamics for two currencies whose supplies vary differently. However, in the long term, that is, regarding trends, the difference could be reduced due to the complex mechanism of the interaction between currencies' trade and central bank activities, so we can catch the differences easier if we're focused on short-term analysis. The descriptions based on (1) and (2) are rude, however. This can be clarified if we remember the dynamics of asset prices in general. In this case, despite the direct effect of cause factors that appear in (1), the market is sensible toward every change in the currency's price. It acts immediately in the next time moment of trading following the Equation 4:

$$x_t = f_t + \alpha \frac{E_t[dx_t]}{dt} \tag{4}$$

where $\frac{E_t[dx_t]}{dt}$ is the expectation change of the logarithmic value and $f_t$ is the fundamental value which is assumed to follow a random walk with volatility $\sigma$ [25]. Parameter $\alpha$ represents the sensitivity of the price $x_t$ to the market exemption changes of it. So, if no TZ is implied, $E_t[dx_t] = 0$ and $x_t = f_t$ . Within a random noise term, fundamental vale and nominal value given in (1) represents the same quantity. Sensitivity parameters $\alpha_1$ for the foreign currency (1) that has a lower cost of transaction due to its use in the role of national currency would differ from sensitivity $\alpha_2$ for another foreign currency. In the simplest scenarios that is if no interventions are made from the outside (Central Bank), the evolution of the exchange rates as function of the nominal value which plays the role of the fundamental is obtained in Bertola et al. [26], by the relationship $x_i(f_i) = f_i + a_i v_i + A_{1i} e^{(\lambda_{1,i} f_i)} + A_{2i} e^{\lambda_{2,i} f_i}$ where $\lambda_{1*}$ and $\lambda_{2*}$ are to constants appearing after integrating stochastic Langevin Equation 5 and $\lambda_1 > 0$ and $\lambda_1 < 0$, [26]. Because the constants are different, once again, the difference in the dynamics between two series of exchange rates is apparent. If money policies are applied, like in the Target Zone model [27–29], the dynamics of (2) become even more complicated. Even though this is not a clear TZ policy for the currency trade in the country, the model is helpful to understand that he expected a significant difference between the short-term dynamics of the currency under scrutiny. For this scenario, a better picture of the dynamics of exchange rates (x) can be provided by considering a more realistic relationship proposed by Lera et al. [25].

$$x_t = f + \log\left(1 + a\frac{E_t[dx_t]}{dt} + O\left(\left[\frac{E_t[dx_t]}{dt}\right]^2\right)\right) \tag{5}$$

where $a = \frac{2}{\sigma^2}\frac{1}{1+\theta(\theta-1)}$, $\sigma$ represent the volatility of the fundamental (f) and $\theta$ is another parameter related to the fundamental volatility, domestic (r) and foreign ($\delta$) risk-free rate given by $\theta = \frac{\delta - r + \frac{\sigma^2}{2} - \sqrt{2r\sigma^2 + \left(\delta - r + \frac{\sigma^2}{2}\right)^2}}{\sigma^2}$. The presence of the informal currency in the market, and the use of foreign currency for direct payment bypassing the conversions, would affect this last parameter substantially. In this regard, stochastic Equation 6 reveals substantial nonlinearities of the dynamics initiated by a simple money supply shift of the type ascribed in (2), which initially is expected to impose a shift on the FX rate $x_t$according to a naïve application of (1) and (3). More generally, the sensitivity parameter $(\alpha)$ in (5) and (6) would be conditioned by them, resulting in different pathways for the spot prices $x_{t,i}$. Based on this descriptive analysis, in Bollen et al. [7], we have discussed the dissimilarities of the local behaviors for Euro-ALL FX rates and USD-ALL FX rates. Herein, we will follow this idea by exploring statistical features related to the suspected effect of the hidden variable, the use of the euro as a national currency substitute. The dynamics can apparently be different even in ideal, risk-free scenarios. So, according to [25, 26] if domestic and foreign risk-free rates are equal for each of the currencies, we have $\theta_{1,2} = \frac{1}{2} - \frac{1}{2}\sqrt{1 + \frac{8r_{1,2}}{\sigma_{1,2}^2}}$ and the sensibility toward changes would be:

$$\alpha_{1,2} = \frac{1}{r_{1,2} + \sigma_{1,2}^2} \tag{6}$$

Putting (6) in stochastic Equations 5 or 6 would result in distinguishable dynamics for the FX of the currency according to the appropriate values of this parameter. It would affect both Bid and Ask values; therefore, studying dissimilarities between their statistical features would provide us with indirect evidence of the remarkable different values of parameters (6). The next step is to explore among dominant cause factors, and we hypothesize that informal use of one of the currencies under analysis (the Euro as Lek substitute herein) is the responsible factor because it is the only one that is permanently present, and the amount of its use in the informal sector is not neglectable, despite being unknown. The most significant effect would be apparent in the analysis of the Open and Close values of the FX rates, which can be a summation of the elementary incremental of the successive changes of the type;

$$\Delta x_t \equiv x_{t+1} - x_t \cong dw + \log \frac{1 + a_{1,2}\frac{E_{t+1}[dx_{t+1}]}{dt}_{1,2}}{1 + a_{1,2}\frac{E_t[dx_t]}{dt}_{1,2}} \tag{7}$$

where $f_{t+1} - \mathrm{f_t} \sim dw$ is a Brownian walk term as by theoretical assumption in Lera & Sornette [25] and Bertola & Caballero [26] or Krugman model [29]. The second term differs by a nonlinear term because the expectations are different in structure and by a multiplicative term. However, we do not have series of Open and Close FX data series, but their differences would have the same nature as Bid and Ask series. Therefore, we expect to recognize statistical similarities in the Bid-Ask behavior too. Notice that, despite the not-declared TZ strategy, based on financial management policies in the country [13, 14], there are several rules and operational activities regarding predefined targets for FX rates for the country, which make evident the presence of the additional perturbing factors considered in the above description. It ensures that qualitatively, the logic used in this section holds in general, and we can use a retrograde argument to interlink dissimilarities (if) observed, with different comportment of our two currencies under

scrutiny in the market. Note also that Equation 2 refers to a continuous approach and could be valid during trading time, whereas the reference for FX spot value is the opening price stated by the Central Bank in the morning trading.

These specifics reaffirm the importance of the Bid Ask spread from a statistical point of view because it mirrors mostly the features of the market variables, including the specific one that we are interested in directly, the informal use of foreign currency. On the other side, when discussing the dynamics of FX rates of main currencies against domestic currency (ALL), the small size of the country's economy reduces the complexity of the statistical analysis because the economies of counterparts in the trade remain unaffected by home economic processes. Also, the economic environment chosen ensures the validity of the common assumption used in the framework of the standard theory of FX dynamics, whose modification we used as our starting point for this approach. Additionally, the two partner trading economies considered herein (EU and USA) are high-income systems, so theoretically, foreign currency inflows toward the country are expected, which modify the effect of import-export balances in the corresponding FX behavior, enforcing a term that is explicitly included in Equations 1 or 2. From this introductory description, it is evident that the effect of the use of foreign currency in direct transactions can be found on the Bid-Ask spread of corresponding FX rates, but the complexity of the processes involved limits their traceability to only statistical means.

## 3. An Overview of Recent Findings Related to This Work

In our recent addresses on the behavior of the FX rates we have explored some specific features of the FX rate time series behavior in general. So, in Bollerslev & Melvin [8] we have obtained that the relative FX rate $r = \frac{FX_{t+1} - FX_t}{FX_t}$ for the period [2008-2019] followed a specific distribution (known as q-Gaussian) that was found remarkably more unstable than corresponding distribution for the USD-ALL FX rates and price of the Gold. For the time dynamics of the Euro-ALL FX rates, we observed that once a local self-organization behavior was initiated, it faded over time as a result of the sufficient amortizing effect of the informal money presence. In Bollen et al. [7], the multifractal properties of FX time series for the main currencies traded in the country have been scrutinized. It resulted in the multifractal power of the FX rates Euro-ALL (spot values) exhibits a specific pattern that can be related to higher heterogeneity. However, the variance of the relative FX spot values was found to be smaller for those series. When discussing potential factors responsible for those 'contradictory' features, we considered indirect evidence of the informal money presence by comparing calculated values of the informal economy fraction using two models, one based on the fast-moving variables (DGE-Dynamical General Equilibrium) and the other based on slow-changing variables (MIMIC-multiple indicators, multiple causes). The observed difference between those two estimations (varying from 0.2% GDP to 2.8% of the GDP for 2005 and 2017 considered in the original work) is considered evidence of a sizeable informal money in the form of foreign currency, mostly in euros. This amount of money has altered the money's aggregate volumes, which appear in the official databases, and consequently, it imposes adjustments on investment, consumption, stock, etc., which are key variables used in the DGE model.

However, it is also a hidden variable, as it does not appear in the official data used by the MIMIC model [7]. By combining econometric arguments, it is concluded that, behind the latent variable, which is responsible for a remarkable relative change in the estimation of the IE view of the two models, stands the use of the euro as a national currency substitute. In Kovaçi et al. [15], it has been highlighted that even though two crises have hit the country's' economy during 2019–2022, the national currency has resisted and behaved stable, which was unexpected for several scholars and econometrics of the country. It is worth mentioning herein that several actors in the country claim that this behavior related to the national currency is illogical and call for Central Bank intervention. Recently, in the annual addresses of the Central Bank, it has been stated that this institution is ready to use as much as 330 million euros to relax or even stop the depreciation of the Euro and USD against the national currency, if needed. We consider that the estimated amount of Euro to be used by the Central Bank stabilizing intervention confirms indirectly that the amount of Euro used in the informal sector as a direct payment means is sizeable, consistent with our hypothesis elaborated in Bollen et al. [7] and herein.

## 4. Statistical Evidence of Dissimilarities of the Bid and Ask FX Behaviour under the Disbalanced Informal use of the Foreign Currency

The Bid and Asks values of a given FX rate are positioned differently regarding the currencies' market features. So, the Bid value represents the price of the currency that investors are willing to pay [30, 31], or general financial literature for more details, but as we motioned above for our concrete system, it also resumes the interventions of the exterior forces that monitor FX behavior. Factually, those influences are included in the open value of the currency exchange rates at the beginning of the trading period, but it serves as a basis where actors would be oriented and influence them toward market agreement on how much to pay for buying the units of foreign currency. On the other side, the Ask value, which is the price that investors (brokers) would like to sell the unit of foreign currency [30, 32], reflects current market events. The differences Bid-Ask report the market depth and the cost of the transaction. In this regard, each series separately contains specific information regarding the FX environment, factors, and features. The

next various combinations would provide specific statistical or dynamical information about the respective series. Consequently, dissimilarities observed for each of the subcategories would provide evidence for specific effects or factors. When considering the Ask and Bid series separately, we include unavoidably heterogeneities inflicted between the last day's closing value and the current opening one because, between two successive Ask or Bid values, there exists a hidden series of FX values during the parts of the two trading days. By constructing a merged series containing successively $\{bid_{yesterdays}, ask_{todays}.\}$ we believe to gather more information from the heterogeneity and stationary point of view if compared with FX averages. The raw differences of those series contains information for daily variation $\delta_{day} = ask_{todays} - bid_{todays}$ and intraday variation $\delta_{intraday} = Bid_{todays} - Ask_{yesterdays}$, which complement each other. Next, the Bid-Ask spread reports the depth of the market and is an indicator of trading liquidity, based on the correspondence between highly liquid and active markets for small Bid-Ask spreads and less liquid markets for wider Bid-Ask spreads. Therefore, we can use the disproportionality of the Bid-Ask trend to directly evidence similarities in the liquidity market regarding each currency under investigation. In this case, we started with evidence of the dissimilarities, and next, we approached an explanation for the cause factor, supposedly the informal use of foreign currency.

### 4.1. Evidencing Dissimilarities between Euro-ALL and USD-ALL Bid-Ask Time Series

The informal economy would act as a reservoir or source regarding supply-demand dynamics [7, 9]. Once the market starts and the open values are declared for all currencies, the Ask and Bid values for the price of the foreign currency that is used directly in this (informal) sector would be tailored by the additional money supply compensation, aside from the current liquidity misbalances in the formal market. As a result, those series would fluctuate smoother. From a time, series point of view, those fluctuations can be better evidenced in the mixed Bid and Ask FX series. In Figure 1 we have displayed Bid and Ask series for Euro and USD FX rates toward Albanian Lek (ALL) and the raw distribution of their differences $\delta_{daisly} = FX_{Ask} - FX_{Bid}$. By smoothing their respective histograms to visualize key properties, we observe that the fluctuations of the spot prices for each currency apparently differ. The histogram for Euro-ALL exhibits three peaks, whereas the corresponding USD-ALL has two peaks. Also, their raw average spread differs. By using a common smoothing kernel (for illustration, we used the epanenchikov kernel, but it is not important), we observe that histograms of the Bid and Ask values for Euro prices are approached better with a 3-gaussian mixture, whereas for USD prices, we obtain a better fit with a two Gaussian mixture. Based on this empirical decomposition, we have estimated the variances for the mixed series. The values are presented in Table 1. It resulted in mixed prices containing Bid and Ask values for USD-ALL exchange rates exhibiting a larger deviance. Considering also that the price of USD units is usually smaller than that of EURO units [33], this difference is estimated to be significant. We considered it an indicator that a strong factor is acting behind the (classical model) scenes. Next, we admit that it could mirror the different scale of the informal use of the foreign currency, but more arguments are needed. Notice that this effect is not strictly defined or known because it might also interfere with cryptocurrencies, black economy, etc., but we believe that the dominant factor is related to the use of the euro in the informal sector as a direct payment means, following the arguments of Bollen et al. [7], it reduces demand for this currency and the velocity of national currency circulation, introduces heterogeneity as a result of doubling the processes, but reduces FX fluctuations by appropriately modifying the money supply.

**Table 1. Parameters of Gaussian decomposition**

| | Euro-ALL Bid Ask series | | | USD-ALL Bid Ask series | | |
|---|---|---|---|---|---|---|
| | **Proportion** | **Mean** | **Variance** | **Proportion** | **Mean** | **Variance** |
| Gaussian 1 | 0.1837 | 106.25 | 6.9979 | 0.2266 | 107.3825 | 11.7836 |
| Gaussian 2 | 0.3673 | 121.534 | 0.722 | 0.7734 | 119.1607 | 8.5211 |
| Gaussian 3 | 0.449 | 116.554 | 7.5872 | | | |
| | | | 10.3469 | | | 14.5417 |

Since the Bid-Ask spread measures the depth of the market, we have the very first information for substantial dissimilarities between the individual properties of two currencies in the trade. Based on general arguments in the finance literature [17, 34, 35], etc., for example, we argue that the Bid-Ask spread of Euro ALL is living in a lower volatility environment compared with the USD-ALL counterparts. A low-volatility environment indicates an abundance of liquidity in general, and for our FX rates, it indicates a safer medium and a more stable environment for Euro-ALL, which is reflected in a smaller average deviance calculated among constitutive gaussians in the distributions presented in Figure 1 and Table 1. Roughly speaking, the ratio of average deviances is around 35%. It suggests that a good part of it originates from the informal use of the Euro, which reduces the costs of transactions on average, despite other factors that inflict disturbances on the spot values, resulting in three peaks for the histogram. Next, we observe that the depth of the market measured by the row Bid-Ask spread is slightly increasing for both FX rates, with some troubles observed during 2023 (see Figure 2). In this empirical view, we spotted the trend of this

change as another measurable feature that could be compared and analyzed. Basically, the most influencing factors for the trend of the spread Bid-Ask are the export-import balance in each currency and the respective amount of each of them that has reached the economy. Also, another factor is the mutual rapport between two currencies, which imposes more complexity on the analysis. Empirically, we observed that the trends of FXs apparently differed for the period June 2021–June 2022 and showed some similarity for the period 2022–2023. Factually, the exchange rates Euro-USD [33] for this period have shown a decreasing trend of the price of the euro until the end of September 2022, and afterwards the dynamics have changed trend, which obviously is mirrored on the prices of those two currencies in Lek, and next on the corresponding Bid-Ask behavior.

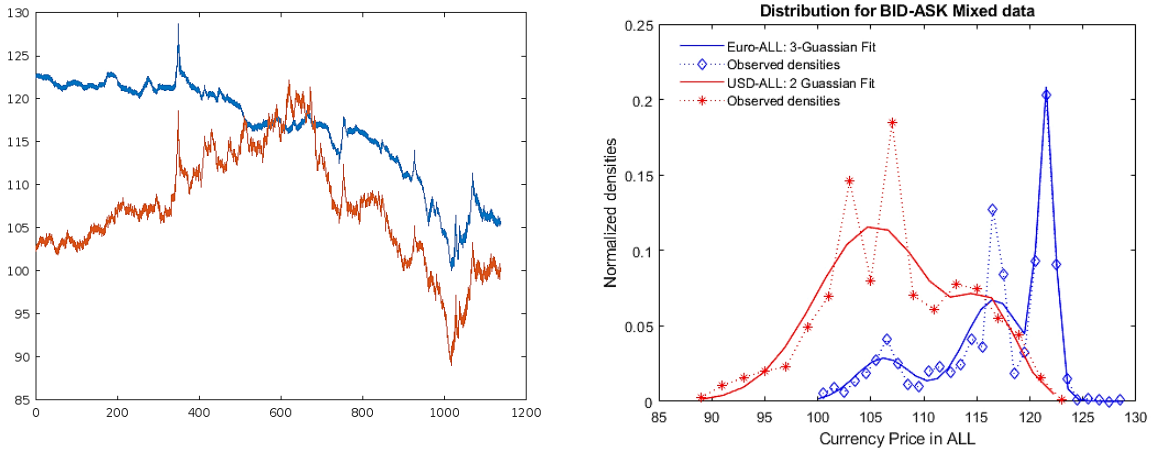


**Figure 1. Composite FX value made up of successive Bid and Asks values. Right frame, distribution shape of the mixed Bid and Ask FX values smoothed by epanenchikov kernel**

To enhance comparing behaviors, we have analyzed relative spread $S_{Relative} = \frac{Spread}{SpotValue}$ (see Figure 2). For a qualitative discussion we assumed a linear trend. Obviously, it changes over time, but even this empirical parameter would show locally the rate of the growth/decrease.

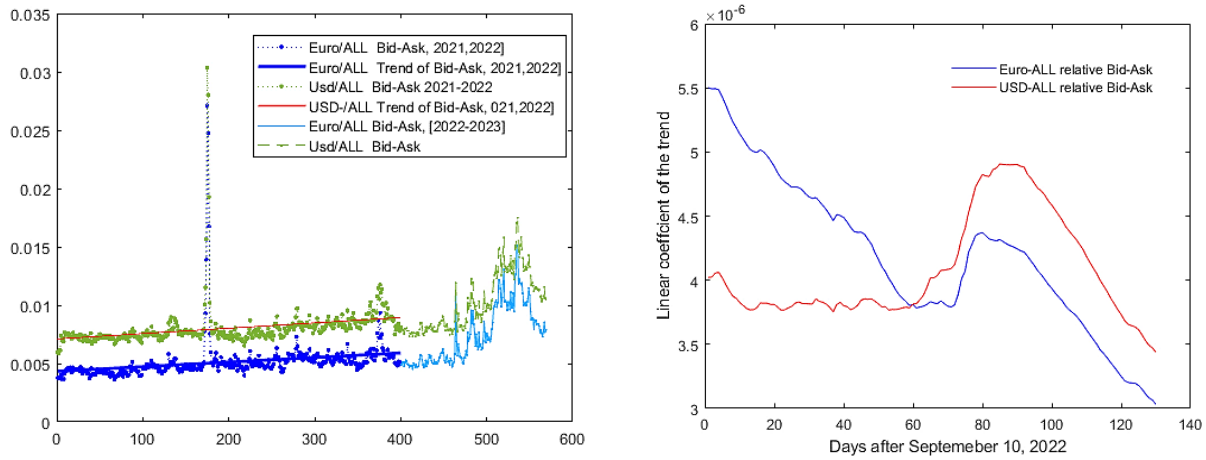


**Figure 2. Plot of Bid-Ask series for Euro/USD-ALL FX rates. A slightly monotone increasing trend is observed. A sharp move of the normalized Bid-Ask is identified in a localized time moment**

Notice that the local properties of the series can be analyzed in the framework of multifractal analysis and the advanced multidisciplinary approach provided in [17, 34], etc., which is not the scope of this study. Some findings regarding this analysis for our series can be found in Bollen et al. [7]. Here we used an empirical linear approach to the trend for relative Bid Ask spread ($denoted\ S_{relative}$) to enable the comparison. Under this assumption, it resulted that $S_{Relative}$ keeps increasing for both FXs, but slower and slower after the coordinate 390 (mid-January 2023). After February 2023 the quasi-linear trend of spread broadening is destroyed for both series (see Figure 2), the right-hand frame. The quasi-monotonic increasing of the trend for both $S_{realtive}$ (or equivalently, the broadening of corresponding FX market depth) signals accumulating economic or financial problem for the country. It can be related to inflationist pressure and import-export imbalances, whereas the increase in the volatility of both series merits more attention regarding the liquidity sector and currency market. Hereto, the behavior of the $S_{realtive}$ for both currencies reaffirms two regimes corresponding to coordinates [1, 400] or June 2021-January 2023, and the other is February 2023-October 2023. Therefore, we chose the relative salient phase covering the time between coordinates [1-400], for

this empirical comparison. The linear coefficient for child series that finishes at a time moment between August 2022, February 2023, is displayed in Figure 2, the right frame. As a result, the rate of the growth of the $S_{relative}$ has changed significantly around the end of September, but this change is specific and local. Considering relative dynamics Euro-USD [33], we argue that it is related to the factors outside our system. The enlarging rate has decreased significantly in a short period of time for Euro Bid-Ask, and it remained nearly constant for USD FX cases. It looks that the currency market has suffered a liquidity difficulty for transactions (or import-export) in Euro, but the market has normalized during October–November 2022, and again another trouble occurred, but the Euro-Lek FX spread has continued to reduce the inherited widening tendency as displayed by the blue curves in the right frame of Figure 2. So, the Euro FX Bid-Ask spread has regained 'good position' more quickly than the spread of USD FX Bid Ask values.

These findings signal that a domestic factor has helped in the reduction of the initial growth of Bid Ask spread of Euro-ALL FX, which could be assumed to be a result of common factors in both currencies' markets. The factor behind this dissimilarity reveals relaxing features on the market difficulties, opposite to them, that cause the Bid Ask Spread to increase. Bearing in mind the dependency of the Bid-Ask spread on the cost of the currency transactions according to the general arguments provided in [30, 35], this fact suggests that the imposed uneasiness of the exchange operations has relaxed over time, which supports the idea that the factor causing this behavior is permanently present in the system. From a general picture of the overall comportment of the currency market and country's economy, and by combining those findings empirically, we have qualified the use of the Euro in the informal sector in the role of the national currency as one of the factors that reduced the cost of the transactions of this currency in the domestic FX market. The only unanswered question for this conclusion reminds the weight of this effect. It is an unknown part of the game because we do not know the size of the Euro used directly in the informal sector as payment means, e.g., national currency substitute. For estimating it, let's consider the corresponding fraction of the transaction volumes, p%. The informal economy is calculated at around 30%–32% [10, 11], and by our own estimation using DIMIMC, it goes at 33%–36% [7]. Regardless of the figures, around 30*p% of the transactions are made in euros, which is assumed to impose a significant reduction in the costs of trading euros in the internal market. However, lacking quantitative assessment by a certified model, we consider the findings of this paragraph supportive but not conclusive on the idea that the dissimilarities observed are related to the use of the Euro in the informal market in the role of the domestic currency. For a better understanding of those elements, we will consider in the following section the averaged behavior of the spread based on the distribution properties.

### 4.2. Features of the Distribution Euro-ALL and USD-ALL FX and Their Bid-Ask Spread

Using the distributions for analyzing the global features of the variables is a common practice in descriptive analysis. It helps to estimate average values $x_{average} = \int pdf(x)xdx$, standard deviation and other moments similarly, and to understand conditions when those important parameters cannot be evaluated at all, etc. We also consider their properties for identifying dominant processes that govern data series generations. In this regard, an intriguing pdf function is the q-gaussian introduced in [36, 37], etc., to ascribe the distribution of certain variables in the out-of-equilibrium physical states. Specifically, it can help to analyze the un-stationarity degree of the distributions, which is another apparent feature that we propose to use for comparison, following the discussion of Kushta et al. [38]. It has the form $G_{q(x)} = \frac{1}{\sqrt{2}\sigma_q Z_q}\left(1-(1-q)\left(\frac{x-\mu_q}{2\sigma_q}\right)^2\right)_+^{\frac{1}{1-q}}$, [36, 37, 39]. The parameters appearing in this functional form such as the statistical integral $z_q$, q-standard deviation $\sigma_q$, q-mean e $\mu_q$, are defined and calculated after q-entropy optimization and in the framework of the q-central limit theorem [37]. Notice that q-Gaussian is typically a t-student distribution with $v = \frac{3-q}{2-q}$ [37, 40], and better describes the fat tails of real histograms in financial analysis [36, 39], which aligns with our desire to analyze the entire range of values for our variables of interest. But the most important feature regarding our interest is that the distance from the stationarity is measured directly by the q-parameter via $d = q - 1$ [36]. The distribution is stationary only if $q < \frac{5}{3}$. For analyzing stationarity features and heterogeneity issues, we have performed a careful procedure for identifying the core distribution of our variables. Initially, we used a standard method for histogram optimization on nonlinear series, e.g., the Freedman Djaconic, Struggle, Soctt, and Nlog methods. We discovered that in several cases, many bins were empty, followed by one or a few occurrences of very high amplitudes (so that a few extremal bins were populated). For those incidences, we assumed next that they are probably related to exterior influences, so, we have qualified the best optimal bins size the one that guarantee smallest number of empty bins subsequently. This procedure is carried out empirically herein, but the logic behind it is based on the idea that we want to know and compare the most apparent features, and for this argument, the very high amplitudes are accredited as atypical. Initially, we performed a goodness-of-fit check for a set of 17 basic distributions, and next, we picked a few of them that showed the best fit according to the AIC and BIC criteria.

### 4.2.1. Distribution of Relative Spread

The broadening of the FX Bid Ask spread depends on liquidity issues, the cost of the transactions, and information asymmetry [30]. Information asymmetry is important because traders who have access to information can demand a higher premium to trade, so the market itself depends on its traders, that is, those who own the currency. Because there always exists a player who knows more (the Central Bank), by focusing on the Bid-Ask spread, which depends mostly on daytime free trading, we can significantly avoid this influence. Also, we assume that the holders (users) of informal currency might adherently modify the supply-demand balances during this free trade. They increase the fraction of un-informed traders by reducing the asymmetry, which causes the spread to decrease proportionally. Holding currencies with higher informational asymmetries requires a positive risk premium for taking the risk of trading against informed investors [30, 41]. The abundant reserve of foreign currency will obstruct this asymmetry, causing the Bid and Ask to come closer. As a result, the average Bid-Ask values would be smaller for the currency that is used more in the informal market. In this regard, the relative values of the Bid Ask $r_{bid-ask} = \frac{FX_{Bid}(t) - FX_{Ask(t)}}{FX_{Average}(t)}$ would facilitate the comparison of respective market depth currency, so we propose to consider this last for the following analysis. Bearing in mind those expectation and arguments, we have analyzed the potential dissimilarities between two FX spreads by using the best fitted distributions to the current (optimized) histograms. In the stage of identification of characteristic distribution for our parameter, we obtained that General Extreme Value (GEV) and t-LocationScale were usually found as the best-fit pdfs (Figure 3). Regarding the t-LocationScale, which is a member of the t-Student distribution, it follows that it is basically a q-Gaussian type, according to Umarov et al. [37]. The GEV-like distribution advocates for the importance of extreme events in the series, but a more careful investigation revealed that this matching property is evocated by a few very high FX values, which implicate that one or two bins at the most extreme of the histogram were populated, but several preceding them were empty. By removing them, the remining part of the histogram representing more than 98% of the events, we observe that GEV was not among the 3–4 best fitted distributions, and t-LocationScale becomes the best fitted pdf. So, it appears that Bid and Ask values were not governed by extreme-value processes, whereas very high values observed would be related to an exterior or local factor.

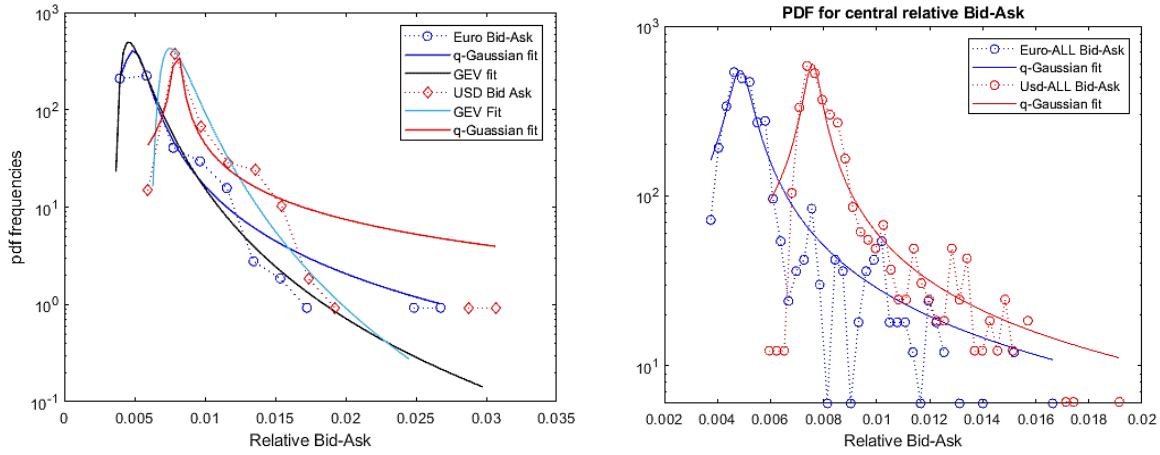


**Figure 3. logarithmic representation of the distributions fitted to the relative Bid-Ask histograms Left frame, GEV and q-Gaussian fit. Right frame, Distributions for shortened histograms**

Based on this evidence for the q-Gaussian nature of the underlying distribution, we may switch to the q-Gaussian form to estimate the unstationarity degree. Notice that q-Gaussian has been classified as a very powerful tool for describing the distribution of financial indexes and similar variables [36, 37, 39], which reinforces our proposal to disregard the last points of the real histogram as not typical. Herein, we obtained that the series of Euro-ALL Bid Ask spreads have a better goodness of fit with the q-gaussian compared with the USD-ALL spread, but the q-parameter is higher. This finding advocates the presence of multiple processes in the formation of the Bid Ask for Euro-ALL FX rates; some of them impose un-stationarity, but the others provide a dominant rule on variable values, which is mirrored in the better-fitting features. To explore those behaviors quantitatively, we used the optimal bin obtained by our ad hoc criteria, e.g., "minimal empty cells.". The fit has improved in favor of q-gaussian, but the q-parameter resulted high again corresponding to un-stationary zone, $q \sim 2.03$ for $r_{bid-ask}$ Euro-ALL and $q \sim 1.66$ for $r_{bid-ask}^{\mathrm{USD-ALL}}$, respectively. Notice that for fitting procedures, we used Nonlinear Least Square (NLLSQ) method combined by the Particle Swarm Optimization (PSO) algorithm, written in MATLAB. Here, when limited in the central part of the histograms (by removing the last bins), the fit by NLS and PSO gave the same results (parameters matched), confirming indirectly that the very high FX values were not of the same nature with other FX for both the Bid and Ask series. Following this procedure, we have obtained $q \sim 1.92$ for $r_{bid-ask}^{Euro-ALL}$ and $q \sim 1.64$ for $r_{bid-ask}^{\mathrm{USD-ALL}}$,

hence, the stationarity indicator for the distributions differs remarkably. It resulted that $r_{bid-ask}^{Euro-ALL}$ are more un-stationary but their average deviance is smaller than the USD-ALL relative spread. The result is interesting. The average mean of the relative spread, which is also the indicator of the market's depth for USD-ALL exchange trade is twofold higher than corresponding EURO-ALL case. Their deviances are comparable, with slight extension for USD-ALL FX series that become remarkable if considering relative term ($\frac{\sigma(spread)}{<spread}$). This indicates that some factors have contributed to stretching the daily Bid Ask differences for the FX rates in Euro-All. It can be achieved by lowering the cost of the transaction, improving liquidities, decreasing asymmetries, etc. Again, we don't identify other factors that can differentiate those behaviors except the one that we have suggested in the beginning of this paragraph, empirically. By resuming those findings, we admit that the use of Euro in the role of Lek contributes to the reduction of transition costs, and the reservoir/source behavior of the informal use of foreign currency in the market are directly cause factors for this set of properties explored herein.

### 4.2.2. Distribution for Bid and Ask Differences.

Bid/Ask series separately can be considered as outputs of some dominant factors on fixing FX spot values. Terms are interrelated, however. The current depth of the market $\delta_{day} = ask_{todays} - bid_{todays}$ that is the FX spread, has its effect on the succeeding value, that is, $bid_{today} \sim spread_{yesterday}$. Those dependencies would degrade the individuality of each series, Bid and Ask, separately. However, bearing in mind that Ask, Bid, and Average FX values are cumulative results, we accept that, on average, those series embody typical dominant processes that survive this mixture. Therefore, by analyzing the distribution of their fluctuations separately, we can gather valuable information about the dynamics of the series themselves. Firstly, we observed that the distributions of Bid and Ask fluctuations matched only in the central zone but differed for high values (see Figure 4). It is not a remarkable novelty because we mentioned above that Bid values contain exterior effects as they are closely related to open values. But the rate of their change or successive differences can tell more. We observed that the standard Fredman-Djaconic optimization procedures based on the interquartile identify remarkable changes in the daily variances of Bid and Ask values for Euro-ALL FX rates. It is recognized directly by the optimal bin-number which is found $n_{bins} = 50$ for Bid series and it reduces to $n_{bins} = 26$ for the Ask series. This is not observed for corresponding USD-ALL rates.

**Table 2. Statistical parameters for q-Distributions fitted to Euro and USD-ALL Bid and Ask series**

| | Zq | $Stdev_q$ | q | q-mean | Goodness of fit (NMSE) | mean | std | $R^2$ | RMSE | df |
|---|---|---|---|---|---|---|---|---|---|---|
| Euro-ALL Ask | 2.79 | 0.12 | 1.99 | 0.00 | 0.05 | 0.38 | -0.03 | 0.95 | 0.11 | 54.00 |
| Euro-ALL Bd | 2.49 | 0.13 | 2.06 | 0.00 | 0.05 | 0.43 | -0.03 | 0.95 | 0.10 | 30.00 |
| USD-ALL Ask | 0.76 | 0.61 | 1.46 | 0.01 | 0.01 | 0.70 | 0.00 | 0.99 | 0.02 | 32.00 |
| USD-ALL Bid | 0.74 | 0.64 | 1.38 | 0.00 | 0.01 | 0.73 | 0.00 | 0.99 | 0.03 | 35.00 |

We observe that q-variance and real variance are nearly the same, indicating that despite high values of the q-parameter which indicate high un-stationarity level of the corresponding distribution, we might use them for semi-quantitative discussion. Remember that quantitative assessment based on the q-parameters of the q-Gaussian is meaningful only for $q < \frac{5}{3}$, that is, stationary distribution. From Table 2 we see that both differences of Bid and Ask series for the Euro-ALL FX are highly nonstationary, whereas for USD-ALL case they are stationary ($q < \frac{5}{3}$). It constitutes a significant dissimilarity which reaffirms the above claim that in the formation of the Bid and Ask values for Euro spot prices, there are present different processes of different nature. We urge that this is the effect of informal use of the Euro that interferes and imposes the un-stationarity. It causes a high relative broadening of the distribution for Euro FX rates, as seen by values $\frac{\sigma(bid,ask)}{mean(bid,ask)_{Euro}} \sim 0.08; 0.07$ that are higher than the corresponding USD FX's, at $0.0062; 0.047$ respectively. Also, the differences of Ask values look little bit less stationary for Euro-ALL FX than its corresponding Bid values ($q_{bid} \sim 1.984, q_{ask} \sim 2.06$). This behavior is opposite for USD-ALL FX rates whose distribution's parameters are $q_{ask} \sim 1.38, q_{bid} \sim 1.46$, supporting the idea that those series are typically different. We claimed that informality issues were the main factor in this observed feature. It is obvious that other exterior factors have contributed to the increase of the un-stationarity level, but their gravity can be comparable for both currencies even if they are not equal, so the main part of the difference in this feature should be attributed to the extensive use of the Euro in an un-natural role, that is, replacing national currency in direct transactions.

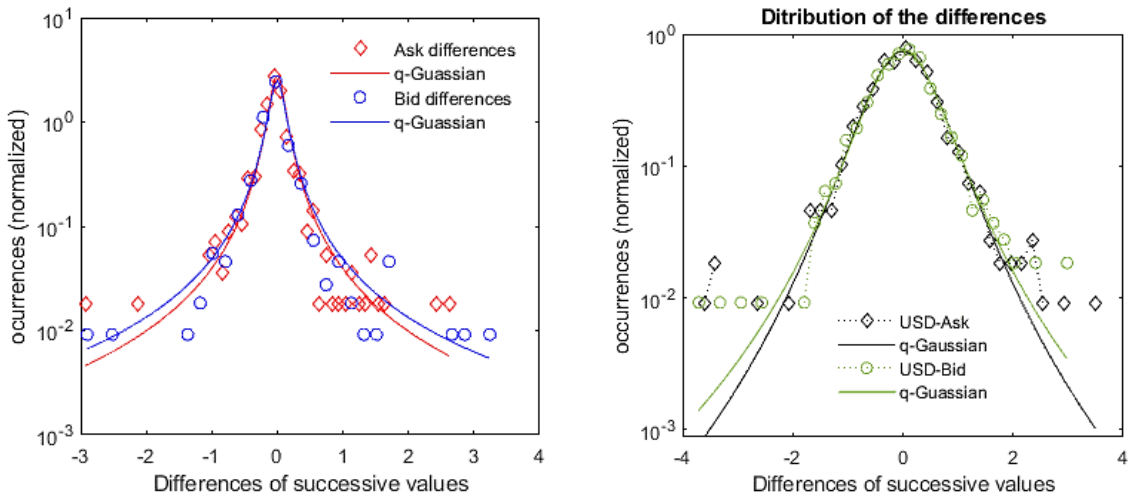


**Figure 4. Distribution for daily differences of Bid and Ask FX values. In the frame on the left, Euro Bid, Ask, in the right, USD Bid Ask. Dissimilarities are observed for every pair of data**

For evidence that those dissimilarities are realistic, we have considered an autoregressive modeling approach for the above variables, but in relative form to facilitate the comparison. We have explored the approach of the GARCH (n, m) model for our series, and we have obtained that GARCH (1,1) is the best model fitted for Bid series of USD-ALL FX rates. However, we observed a good fit with the EGARCH (1,1) model for the corresponding series of Euro-ALL FX rates. Based on those exploratory observations, we supposed that the reproduction of the distributions of the return for current series could be an estimator of the heterogeneity that characterizes each series. In Bollen et al. [7], this diagnosis was conducted using multifractal analysis, which confirmed a higher heterogeneity measure for the series of Euro-ALL FX rates compared to USD-ALL FX rates. So, it resulted in a simulation of the returns for USD-ALL Bid values using EGARCH (1,1) that reproduced the current distribution (see Figure 5).

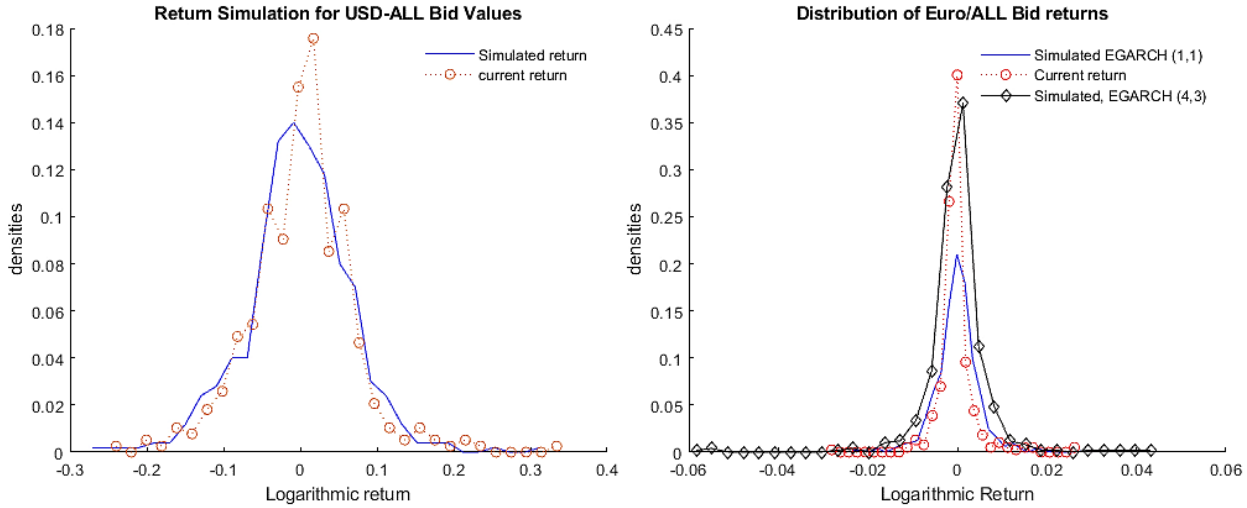


**Figure 5. Distribution of return and simulation using GARCH and EGARCH models**

However, the distribution of the returns for Euro-ALL Bid values of the FX rates is reproduced better by the EGARCH (4,3) model, which nonetheless does not have good p-values. It confirms a higher non-stationarity (and non-linearity) in those series and accentuates the dissimilarity in this regard. We considered this finding as another evidence of the effect of a hidden factor that makes the Euro-ALL return more un-predictable compared with the USD-ALL return.

## 5. A Discussion about the Effect of the Informal use of Foreign Currency

Herein we propose a rude estimating of the effect of informal use of foreign currency in the Bid Ask FX spread, by expanding the indirect investigation procedure used above. Let's consider the most important indicator of the currency market, the spread Bid-Ask. Some assessment regarding modeling the spread can be realized by using BSW model for spread dynamics $S_i = a_0 + a_1 \times d_i + a_2 IHP_i + a_3 InvQ + u_I$ [31] providing that we know variables of the rhs herein. Unfortunately, dealers $d_i$, inventory holding premium IHP, and the inverse of the number quotes InvQ are unknown for our system because we have administered only daily data for Bid and Ask values. But this linear model suggest

that other linear forms can be used, starting from $\Delta S_t = a_1 * \Delta d_t + a_2 \Delta IHP_t + a_3 \Delta InvQ + \varepsilon_I$, we may suggest that dealers are related with an exogenous variable and in principle we can propose the following ad hoc VARX model;

$$\begin{pmatrix} \Delta S_{EURO} \\ \Delta S_{USD} \end{pmatrix}_t \sim B * \Delta Exogenous + \sum_{i=1}^{p} C_i \begin{pmatrix} \Delta S_{EURO} \\ \Delta S_{USD} \end{pmatrix}_{t-i} + u_t \quad (8)$$

Notice that in the role of exogenous variables could be any variable mentioned involved in the models mentioned in this section or in {FX, Price} dynamics. Here we employed the interest rates (overnight interest rates ONIR) as one possible response but also it can be used as predictor, since we are interested in observing dissimilarities and explaining a basis for them, rather than assessing parameters. Next, the informal economy measure (IE) can be used as predictor variable in the standard VARX model. However, we do not have a measure for the volume of foreign currencies used as a substitute for the Lek in the informal sector. To overcome this, empirically we proposed an ad hoc variable to mimic informal economy. It is taken herein as $IE \sim (1 - V_{Normalised})$ where V is the volume of daily transaction in the banking system. It is an income measure and therefore it makes sense, and the difference seeks to catch the out of bank activities (in a very rude estimation). The purpose of this calculation is to evidence and compare the reproduction of the dynamics by using autoregressive models in the presence or without our exogenous variable. So, initially we have spanned VAR (*) models for our series to obtain the most appropriate one that reproduces better the distributions of the FX spreads. Remember that in VAR (n, p) model, n is the number of responses depending on the number (n) of response variables used in (4). Here, for n=3 we obtained p=4 and for n=2 we obtained p=3.

We observed that a series of differences in spread ($\Delta S_{Euro}$) for Euro-ALL are reproduced better when including our ad hoc informality measure in the VARX model (8), Figure 6. In this case, a 30-day forecast for response parameters showed that the reproduction of the EURO-ALL spread by the model has been better than the reproduction of the USD-ALL spread (see Figure 6). It resulted in the finding that including a quasi-informality parameter in the regressive Equation 8 has improved the reproduction of the EURO FX spread, but it didn't significantly affect the reproduction of the USD-ALL FX spread. We conclude that the series of Euro FX spreads can be better explained if we consider an additional exogenous variable, which is latent. It consists of an indirect, supportive argument that the behavior of the spread in this case is conditioned by the exogenous factor. The fact that this is not observed for USD-ALL confirms our assumption that usually this currency is not used in informal transactions.

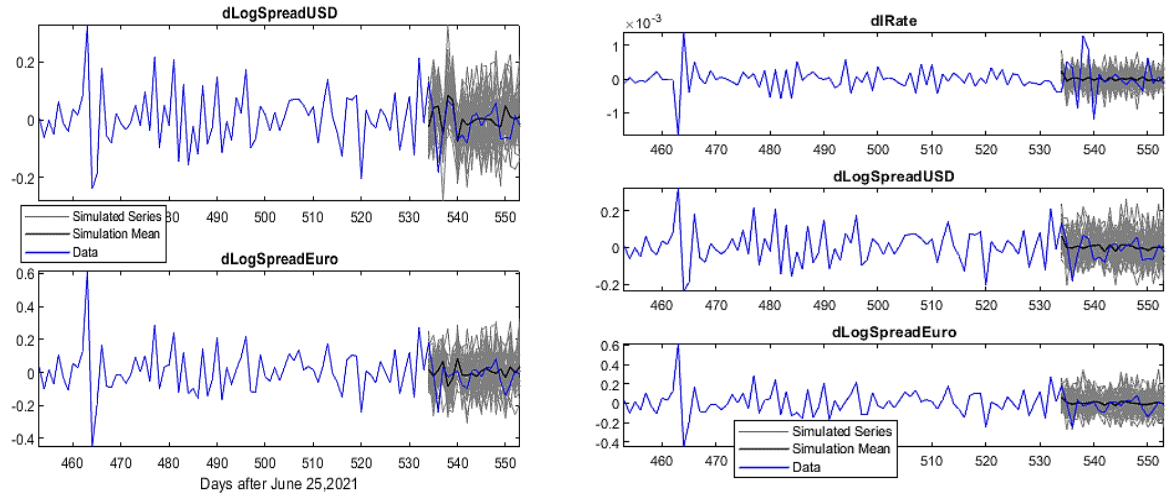


**Figure 6. An ad hoc VARX (2,4) approach for Spread time series and two ad hoc exogenous variables. Left frame, predictors are interest rate and our ad-hoc informality measure, whereas responses are logarithms of the spread. In the right frame, only interest rate variables are taken as exogenous, all others are responses.**

## 6. Conclusion

Finalizing this descriptive analysis, we can confirm that all arguments provided herein clearly certify dissimilarities between the behavior of the USD-ALL and EUOR-ALL FX rates. This behavior persists for all the periods considered, which are chosen not to include specific economic and financial conditions for the country. Our system can be considered more general, providing the achievement of the key prerequisites that the economy must be small and that a certain foreign currency is used in the informal sector as a direct payment means in the case of domestic currency. Next, the level of income can be important, but if two first are achieved, it is very likely that there are inward inflows of foreign currency. Under those conditions, the use of a foreign currency in place of the home currency would reduce the fluctuations of the FX rates for this currency, aside from inflicting perturbations that cause the overall state to be highly non-stationary. If there is a significant inflow of foreign currency into the country, this effect contributes to improved FX performance. If those inputs are not satisfactory enough, this analysis didn't tell

much, but a more generalized view can predict a worsening of the financial situation. This claim is based on the high un-stationarity of the overall state, which can be combined with the high heterogeneity analysis in Guirguis [41] to produce complex behavior. Supplementary arguments can be predicted by an interdisciplinary approach, which identifies hidden energies related to the high heterogeneity states that can be lost if a relaxation path is imposed in the system. A very similar scenario can be developed for a formalization procedure that would be imposed on this system.

Significant losses are expected. It seems to be against the economist's belief that informality can improve economic and financial parameters, but the context should be considered in this case. Firstly, we do not calculate the overall coast of the country's economy for preserving informality at this high level, but we observed that in a special case, it works in a positive way. Second, the use of Euro as an ALL substitute has existed for such a long period in the country. Several real-state operators, construction industries, or touristic entities have avoided the costs of exchanges and used this currency directly. In some way, the market is habituated enough to this practice that it seems quite normal to hear in everyday life the conversion in euros for salaries, wages, and even the square meters of the building for sale, etc. The rapid convergence of the price of the euro toward the value of 100 can promote further extension of the ease of using it directly in common transactions. It seems that the FX exchanging costs are unnecessary for the system, which tries to avoid them if there is no penalty associated with this behavior.

## 7. Declarations

### 7.1. Author Contributions

Conceptualization, D.P. and E.V.; methodology, E.K.; software D.P.; validation, E.K. and D.P.; formal analysis, D.P.; investigation, D.P.; resources, I.D.; data curation, E.V.; writing—original draft preparation, D.P.; writing—review and editing, E.K.; visualization, D.P.; supervision, D.P.; project administration, D.P. All authors have read and agreed to the published version of the manuscript.

### 7.2. Data Availability Statement

The data presented in this study are openly available at: *www.bankofalbania.al*.

### 7.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

### 7.4. Institutional Review Board Statement

Not applicable.

### 7.5. Informed Consent Statement

Not applicable.

### 7.6. Declaration of Competing Interest

The authors declare that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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